Lattice-Boltzmann simulations of turbulent flows

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Fluid turbulence applications in both industrial and environmental topics

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LBM fundamentals

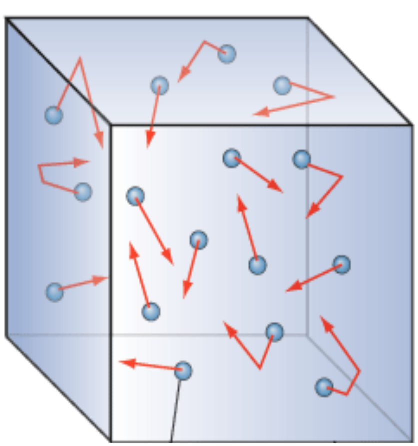
Statistical Mechanics

- Boltzmann Eq.
- Molecules
  - Kinetic energy
  - Momentum
  - Collisions
  - Mean free path

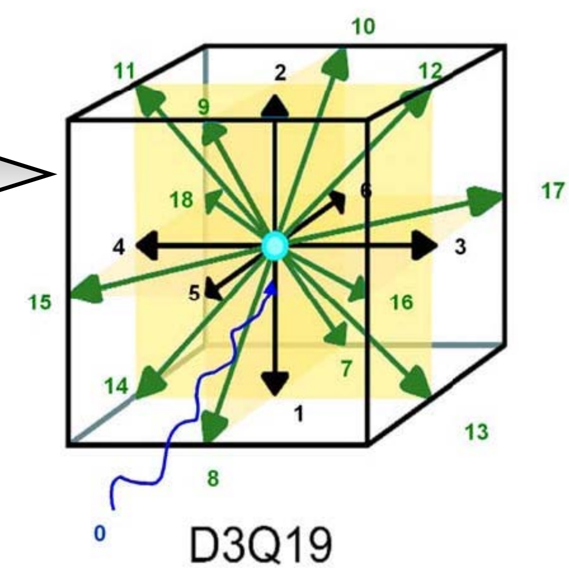
Continuum Mechanics

- Navier-Stokes Eqs.
- Continuous medium
  - Temperature
  - Pressure
  - Density
  - Viscosity

Lattice-Boltzmann



- Volumic mesh
- Discrete velocities
- Probabilistic description



Continuous Boltzmann-BGK equation

f(ξ,x,t)
Single-particule distribution function

External force

Collision term (Bhatnagar-Gross-Krook, 1954)

$$\frac{\partial f}{\partial t} + \vec{\xi} \cdot \nabla f + \vec{g} \cdot \nabla_{\xi} f = -\frac{1}{\tau}\left(f - f^{(0)}\right)$$

velocity

Post-collision Relaxation time

Equilibrium distribution (Maxwell-Boltzmann)

$$f^{(0)} = \frac{\rho}{(2\pi c_T^2)^{D/2}} \exp\left(\frac{-(\vec{\xi} - \vec{u})^2}{2c_T^2}\right)$$

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Macroscopic quantity reconstruction

$$\rho(x,t) \equiv mn(x,t) = m\int f dv$$

$$\rho u(x,t) = m\int f v dv$$

$$\rho e(x,t) = \frac{1}{2}m\int f|\underbrace{v-u}_{c}|^2 dv$$

$$P_{ij} = p\delta_{ij} + \sigma_{ij} = m\int f c_i c_j d\vec{c}$$

Strain tensor

$$Q_{ijk} = m\int f c_i c_j c_k d\vec{c}$$

$$q_i \equiv Q_{ijj} = m\int f c_i c^2 d\vec{c}$$

Heat flux

Regularized LBGK collision model

Distribution function expansion

$$f_i = w_i \sum_{n=0}^{N} \frac{1}{c_s^{2n} n!} \boldsymbol{\mathcal{H}}_i^{(n)} : \boldsymbol{a}$$

First-order approximation

$$f_i = f_i^{(0)} + f_i^{(1)}$$

$$f_i^{(0)} = w_i \sum_{n=0}^{N} \frac{1}{c_s^{2n} n!} \boldsymbol{\mathcal{H}}_i^{(n)} : \boldsymbol{a}_0^{(n)}$$

$$f_i^{(1)} = w_i \sum_{n=0}^{N} \frac{1}{c_s^{2n} n!} \boldsymbol{\mathcal{H}}_i^{(n)} : \boldsymbol{a}_1^{(n)}$$

Malaspinas' recursive rule

$$a_{1,\alpha_1 \ldots \alpha_n}^{(n)} = a_{1,\alpha_1 \ldots \alpha_{n-1}}^{(n-1)} u_{\alpha_n} + \left(u_{\alpha_1} \ldots u_{\alpha_{n-2}} a_{1,\alpha_{n-1}\alpha_n}^{(2)} + \mathrm{perm}(\alpha_n) \right)$$

$$a_{1,\alpha\beta}^{(2)} = -2\rho\tau c_s^2 S_{\alpha\beta}$$

Regularized LBGK collision model

Discrete Velocity Boltzmann Equation

$$\partial_t f_i(\boldsymbol{x},t) + \boldsymbol{\xi}_i \cdot f_i(\boldsymbol{x},t) = \Omega_i$$

$$\Omega = -\frac{1}{\tau}\left(f_i - f_i^{(0)}\right)$$

BGK collision model

$$f_i^{(0)} = w_i \rho \left(1 + \frac{\boldsymbol{\xi}_i \cdot \boldsymbol{u}}{c_s^2} + \frac{1}{2c_s^4}\boldsymbol{\mathcal{H}}_i^{(2)} : \boldsymbol{uu}\right)$$

Truncated Hermite expansion

$$f_i^{(1)} = -\frac{1}{2c_s^4}\boldsymbol{\mathcal{H}}_i^{(2)} : \boldsymbol{P}^{(1)}$$

$$\boldsymbol{\mathcal{H}}_i^{(2)} = \boldsymbol{\xi}_i\boldsymbol{\xi}_i - c_s^2\boldsymbol{I}$$

$$\boldsymbol{P}^{(1)} = \sum_{i=0}^{q-1}\boldsymbol{\mathcal{H}}_i^{(2)}\left(f_i - f_i^{(0)}\right)$$

$$\Omega_i = -\frac{1}{\tau} f_i^{(1)}$$

Hybrid Regularized LBGK collision model

$$\tilde{f}_i^{(1)} = f_i^{(1)}\sigma - (1-\sigma)\frac{\rho\tau}{c_s^2}\boldsymbol{\mathcal{H}}_i^{(2)} : \boldsymbol{S}^{FD}$$

↓

$$\tilde{\boldsymbol{P}}^{(1)} = \boldsymbol{P}^{(1)}\sigma - (1-\sigma)2\rho\tau c_s^2\boldsymbol{S}^{FD}$$

$$\Omega = -\frac{1}{\tau}\tilde{f}_i^{(1)}$$

↓

$$\partial_t f_i + \boldsymbol{\xi}_i \cdot \boldsymbol{\nabla} f_i = -\frac{1}{\tau}\left(f_i^{(1)}\sigma - (1-\sigma)\frac{\rho\tau}{c_s^2}\boldsymbol{\mathcal{H}}_i^{(2)} : \boldsymbol{S}^{FD}\right)$$

↓ Chapman-Enskog expansion

$$\rho\left(\partial_t u_\alpha + u_\beta\partial_\beta u_\alpha\right) = -\partial_\alpha p + 2\partial_\beta(\mu S_{\alpha\beta}) - \left(\frac{(1-\sigma)\Delta x^2}{6\sigma}\right)\partial_\beta(\mu(\partial_\alpha^3 u_\beta + \partial_\beta^3 u_\alpha))$$

[Jacob & Sagaut, JoT, 2018]

HRR-LBM with controled dissipation

Numerical dissipation can be explicitely tuned to recover turbulence model dissipation

Numerical dissipation $\varepsilon_\sigma = \nu_\sigma |\nabla^2 \mathbf{u}|^2 \quad \nu_\sigma = \frac{1-\sigma}{6\sigma} \Delta x^2 c_s^2 \rho \tau$

Equal dissipation $\nu_t |\nabla \mathbf{u}|^2 = \nu_\sigma |\nabla^2 \mathbf{u}|^2$

Artificial viscosity $\nu_\sigma = L_{VK}^2 \nu_t \qquad L_{VK} = |\nabla \mathbf{u}| / |\nabla^2 \mathbf{u}|$

Pseudo von Karman lengthscale

Associated parameter $$\sigma = \frac{1}{6\nu_t \frac{L_{VK}^2}{\Delta x^2 c_s^2 \tau} + 1}$$

HRR-LBM with controled dissipation

Specific case of Subgrid Viscosity Models for LES

General expression

$$\nu_t = \rho c_{sgs} \Delta x^2 / \tau_{sgs}$$

$$\sigma = \frac{1}{6\dfrac{c_{sgs} L_{VK}^2}{c_s^2 \tau \tau_{sgs}} + 1} \qquad c_{sgs} = \left(\frac{1}{\sigma} - 1\right) \frac{c_s^2 \tau \tau_{sgs}}{6 L_{VK}^2}$$

Associated subgrid activity parameter

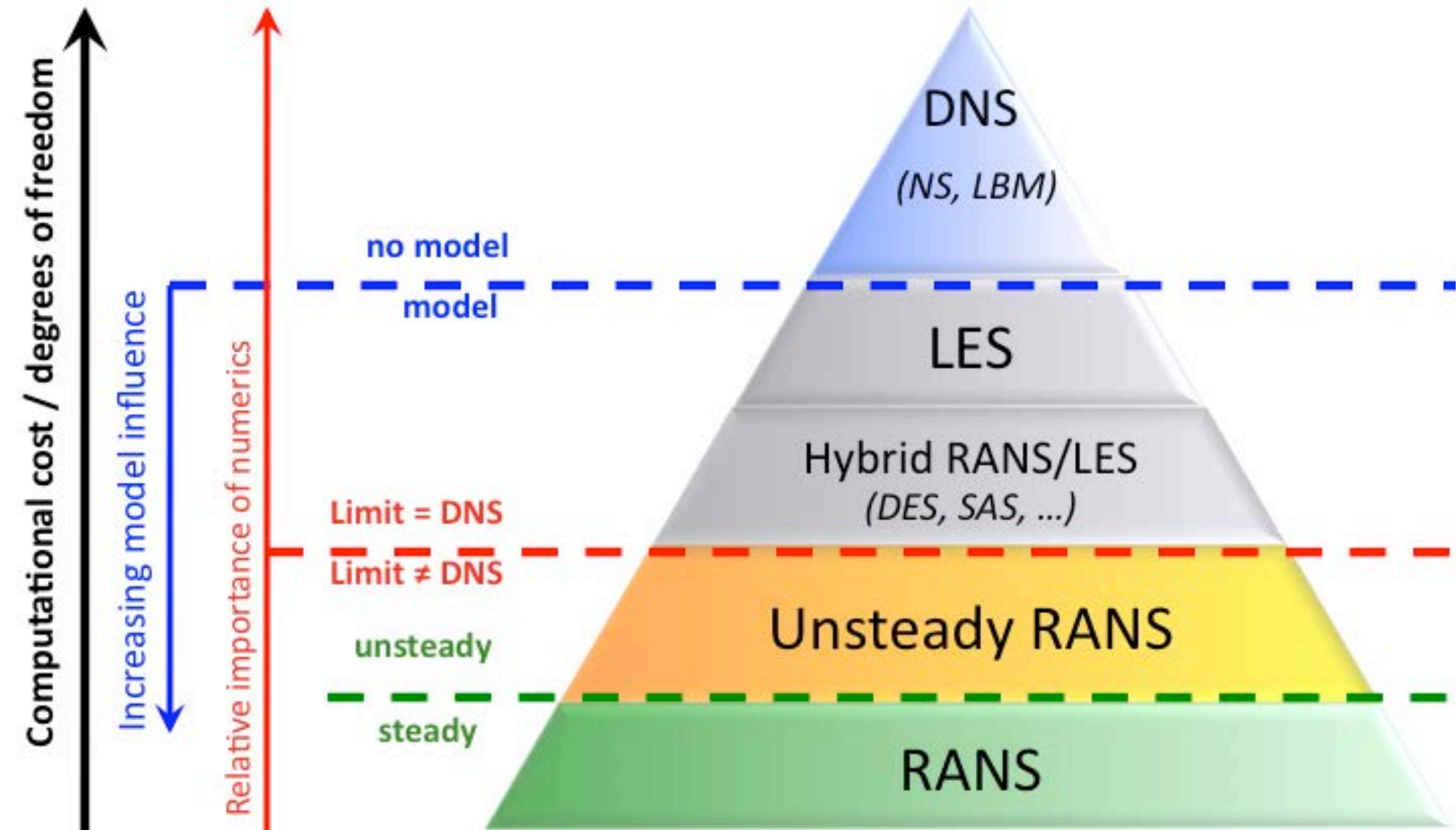
$$s = \frac{\varepsilon_\sigma}{\varepsilon_\sigma + \varepsilon_\nu}$$

s = 0 → DNS
s = 1 → infinite Re

$$\frac{1}{s} = 1 + \frac{6\sigma\nu}{(1-\sigma)\Delta x^2 c_s^2 \tau} L_{VK}^2$$

Hierarchy of CFD methods

« ***Multiscale & Multiresolution approaches for turbulence, 2nd edn*** »
Sagaut, Deck & Terracol, Imperial College Press, 2013



Urban physics for Aerospace engineering

- **Aerodynamics**:
  - Unsteady loads on buildings/aircrafts
  - Wind comfort
  - Realistic wind for flight simulators
- **Heat transfer**:
  - Accurate BC for thermal efficiency models
- **Aeroacoustics**:
  - Wind-induced noise
  - Noise propagation in complex areas
- **Air quality**:
  - Outdoor/indoor pollutant dispersion near airports

Including:
- Micro-meteorological effects (stratification, humidity …)
- Full scale geometry
- Multiphysics couplings, e.g. radiative transfer
- Uncertainty quantification and propagation
- Data Assimilation

LBM-based micrometeorology model

Physical (macroscopic) unknowns:

- Density
- Velocity
- Temperature (absolute, potential, virtual potential)
- Water vapor mass fraction
- Liquid water mass fraction
- Pollutant concentration

Physical mechanisms:

- Gravity → stratification
- Earth rotation →Coriolis effectfs
- Mesoscale (one-way weak coupling only)
- Unsteady turbulent inlet (SEM)
- Evaporation/condensation (volumic balance)
- Vegetal areas (including evapotranspiration)
- Vegetal volumes (trees)
- Ad hoc subgrid and wall models

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LBM-based micrometeorology model

D3Q19 - LBGK
+ Modified Recursive Regularized Collision
+ forcing terms *[Guo et al., 2002]*

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\tilde{u}_i) = 0$$

$$\frac{\partial \bar{\rho}\tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_j\tilde{u}_i) = -\frac{\partial \bar{p}}{\partial x_i} + \left[\frac{g}{\left\langle \tilde{\theta}_{vl} \right\rangle}\left(\tilde{\theta}_{vl} - \left\langle \tilde{\theta}_{vl} \right\rangle\right) - g\right]\delta_{i3} - \varepsilon_{ij3} f(u_{gj} - \tilde{u}_j) + \left(\frac{\partial \tilde{u}_i}{\partial t}\right)_{LS} + \left(\frac{\partial \tilde{u}_i}{\partial t}\right)_{turb}$$

$$\frac{\partial \bar{\rho}\tilde{\theta}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\tilde{\theta}\tilde{u}_i) = \left(\frac{\partial \tilde{\theta}}{\partial t}\right)_{turb} + \left(\frac{\partial \tilde{\theta}}{\partial t}\right)_{cond} + \left(\frac{\partial \tilde{\theta}}{\partial t}\right)_{LS} + \left(\frac{\partial \tilde{\theta}}{\partial t}\right)_{rad}$$

DDF or FD-Hybrid

$$\frac{\partial \bar{\rho}\tilde{q}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\tilde{q}\tilde{u}_i) = \left(\frac{\partial \tilde{q}}{\partial t}\right)_{turb} + \left(\frac{\partial \tilde{q}}{\partial t}\right)_{cond} + \left(\frac{\partial \tilde{q}}{\partial t}\right)_{LS}$$

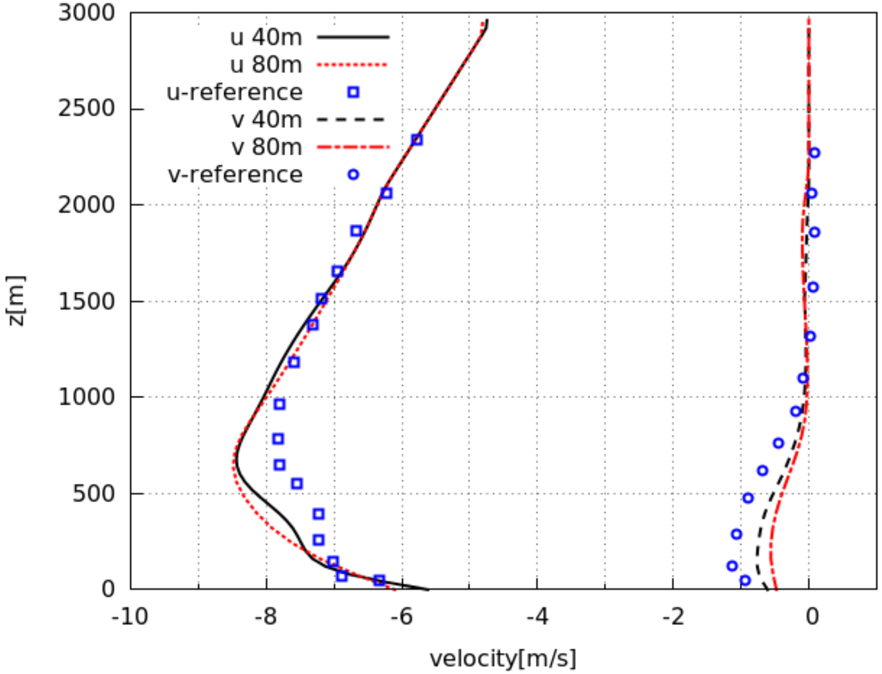
$$\frac{\partial \bar{\rho}\tilde{q}_l}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\tilde{q}_l\tilde{u}_i) = \left(\frac{\partial \tilde{q}_l}{\partial t}\right)_{turb} + \left(\frac{\partial \tilde{q}_l}{\partial t}\right)_{cond} + \left(\frac{\partial \tilde{q}_l}{\partial t}\right)_{LS}$$

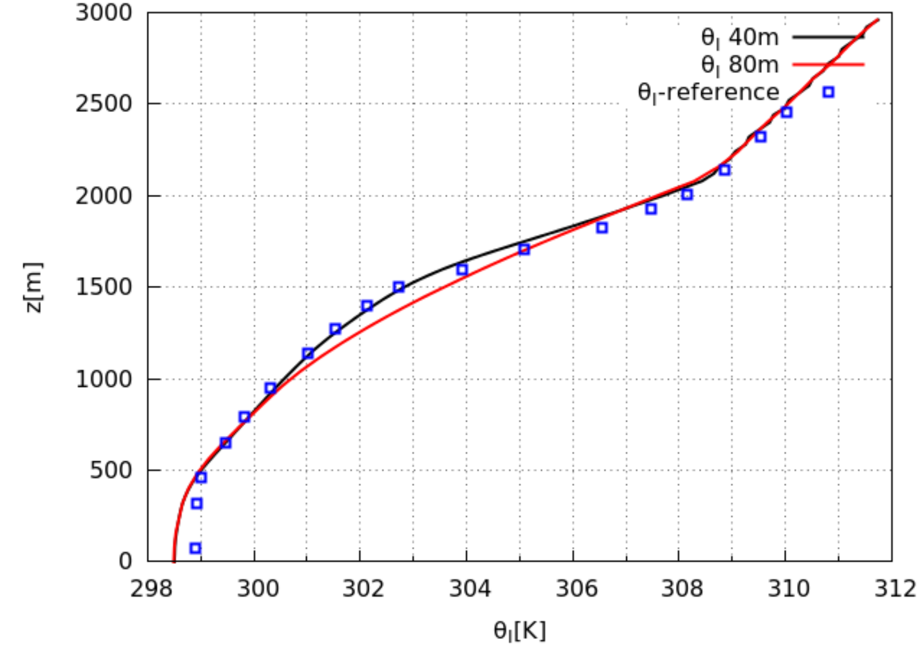
Finite differences

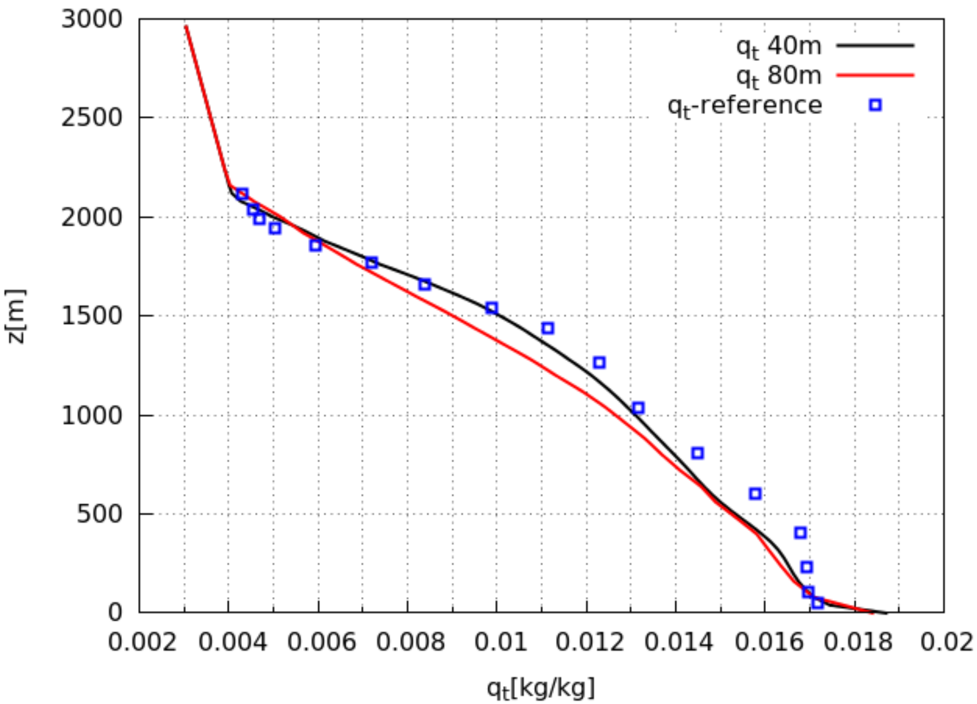
$$\rho RT = P(1 - 0.61q)(1 + q_l)$$

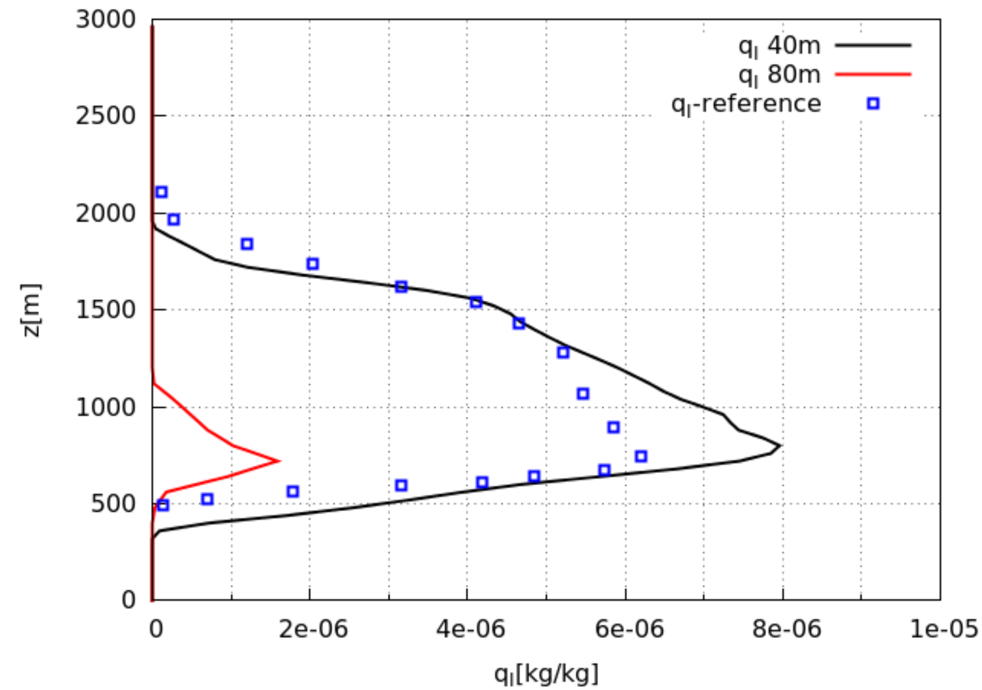
Humid air EOS

Validation: ABL with cumulus clouds









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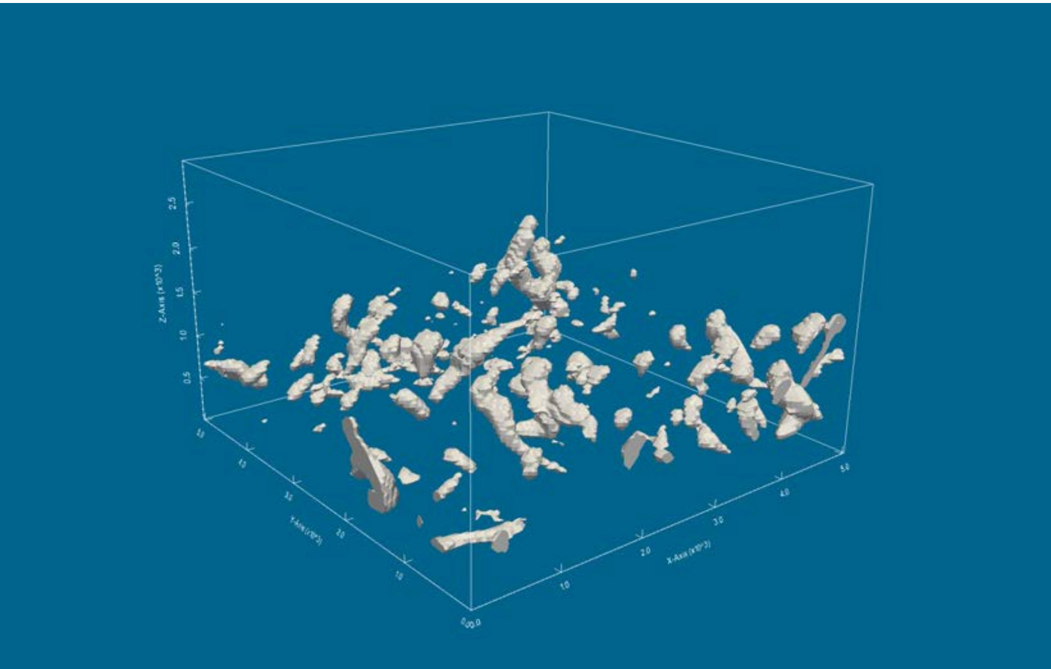
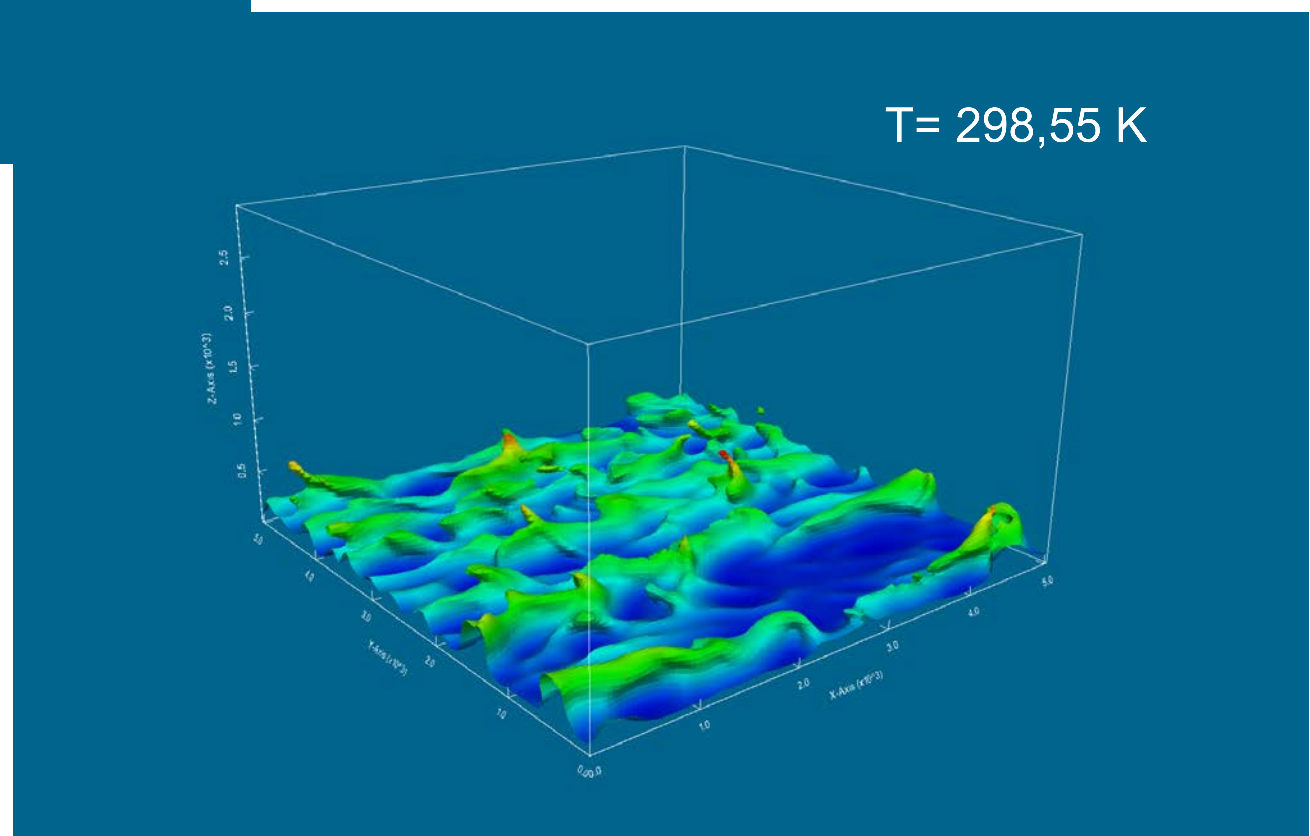
LBM-based micrometeorology model

Liquid water iso-surface at t= 9 hrs



Potential temperature at t= 9 hrs

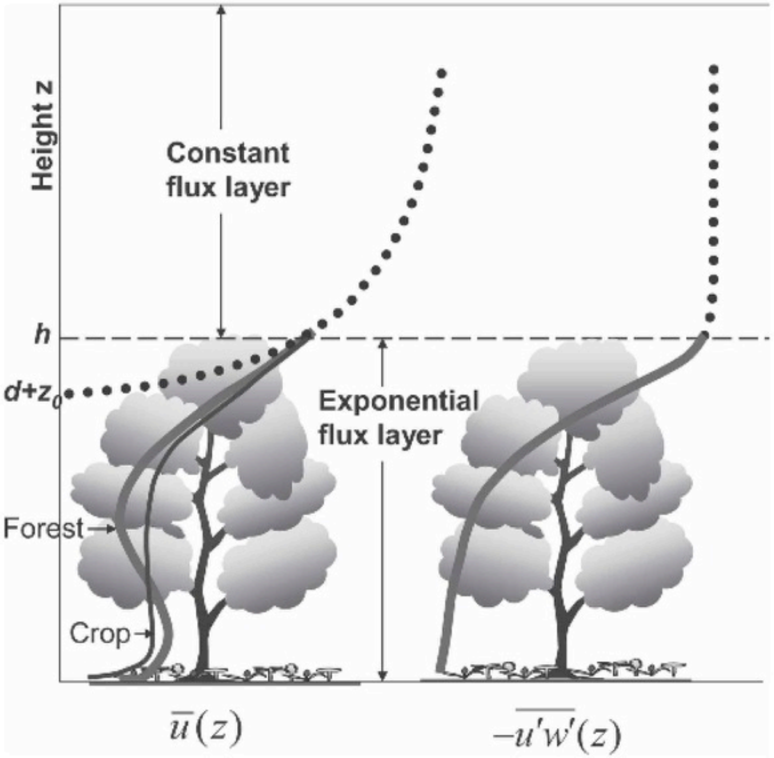


Validation: ABL over forest canopee

$$\frac{\partial u_i}{\partial t} = ... - c_d.LAD(z).|\mathbf{u}|u_i$$

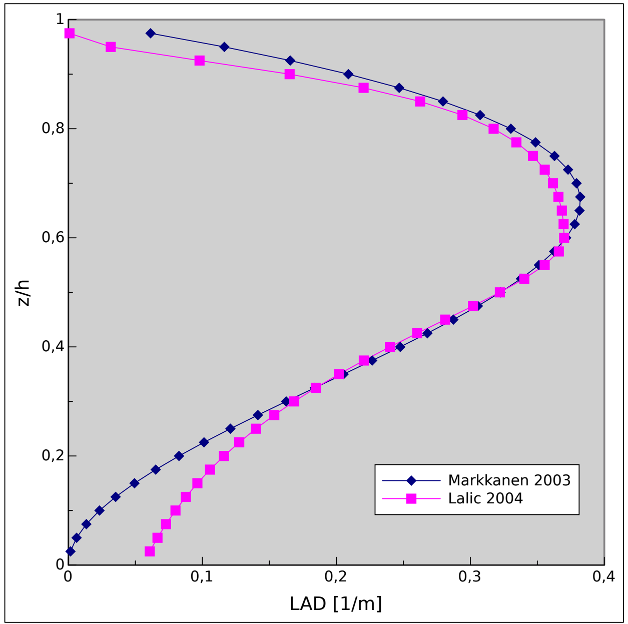
$$\frac{\partial \theta}{\partial t} = ... + \frac{d}{dz}\left(Q_h \exp\left[-\varepsilon_c \int_z^h LAD(z')dz'\right]\right)$$

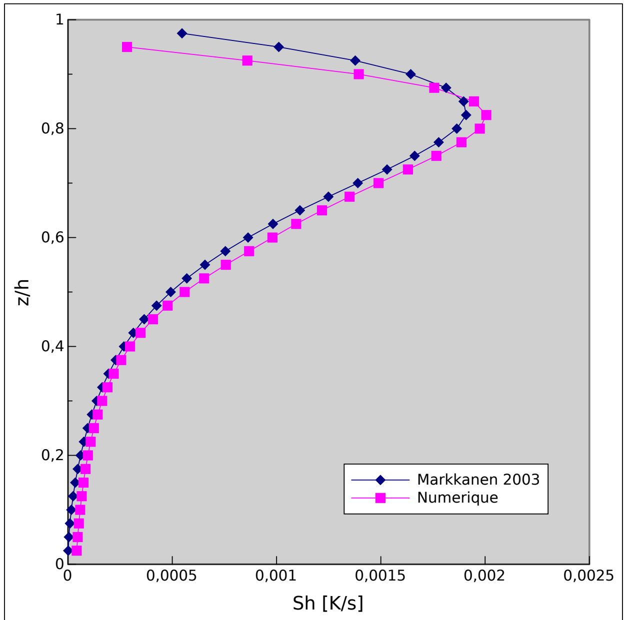
LAD(z)



Z=h (tree top)

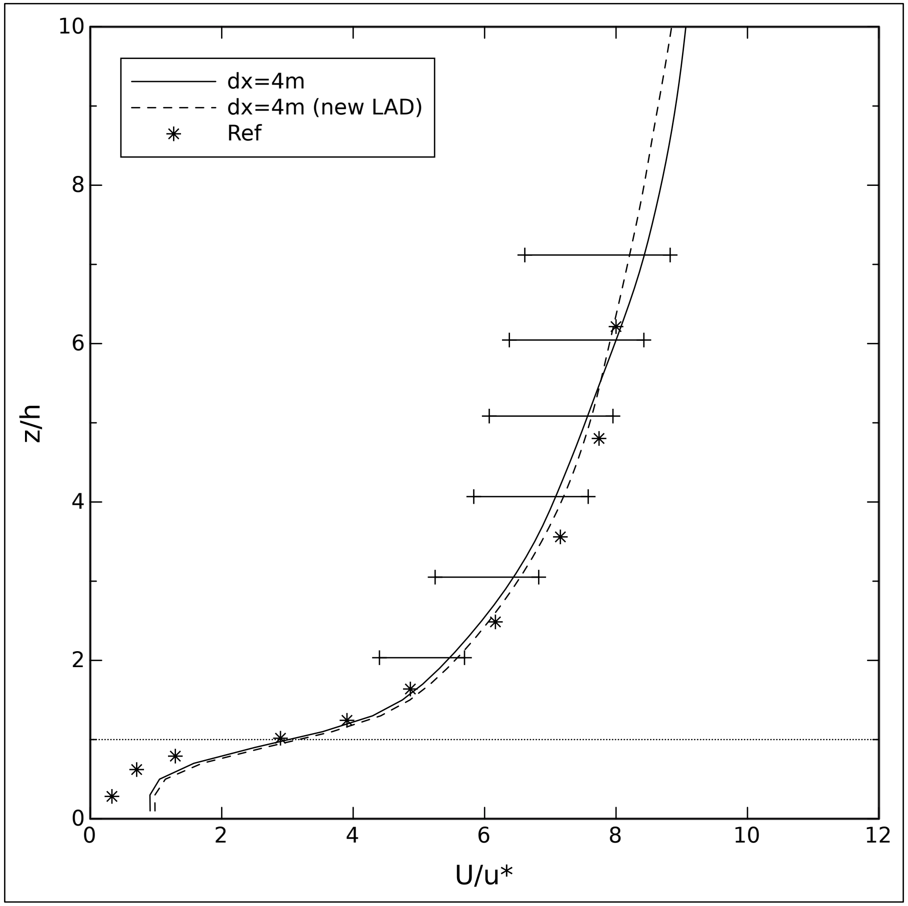
z=0 (ground)

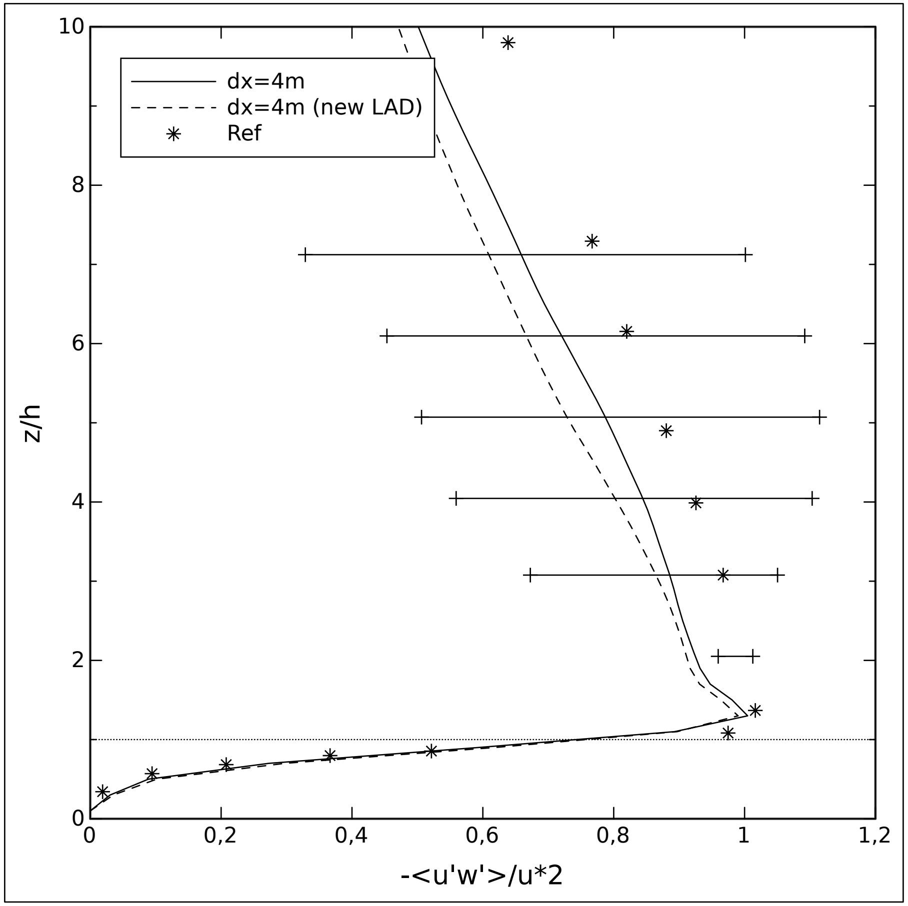




Validation: ABL over forest canopee

Convective BL: $Q_h = 0.015\,K\,m/s$





Domain: 400 m x 400m x 400 m

Tree height: h=20m

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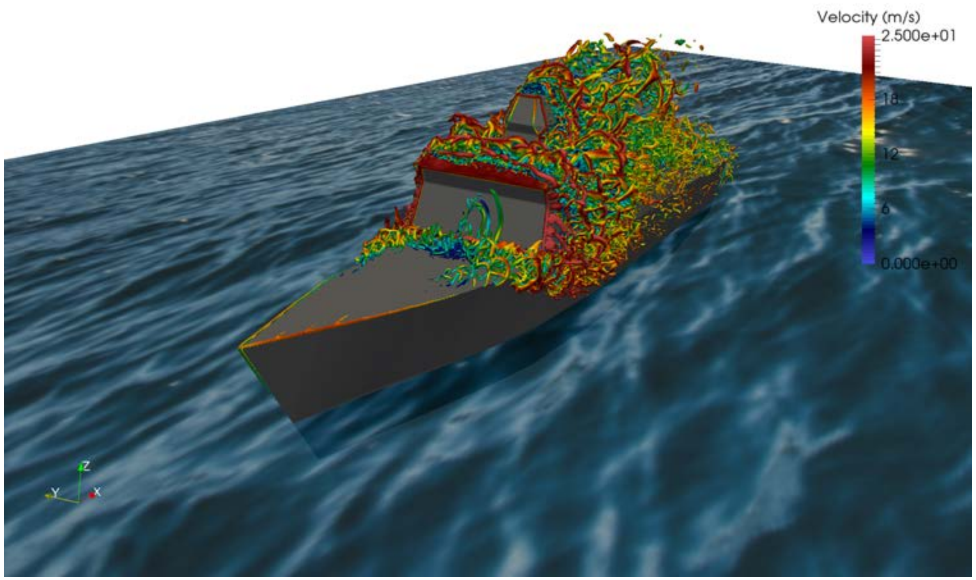
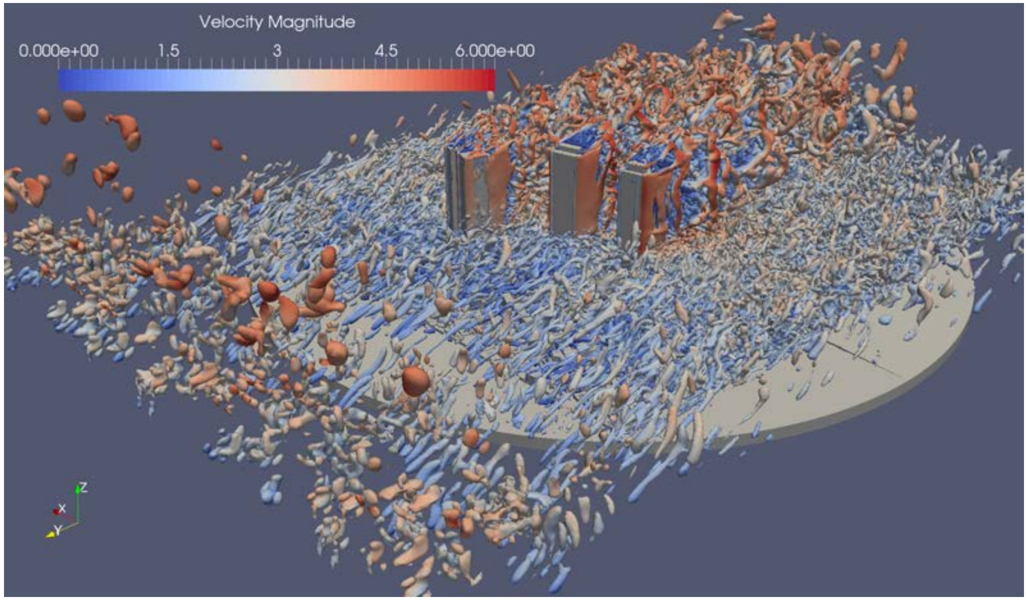
Examples of application

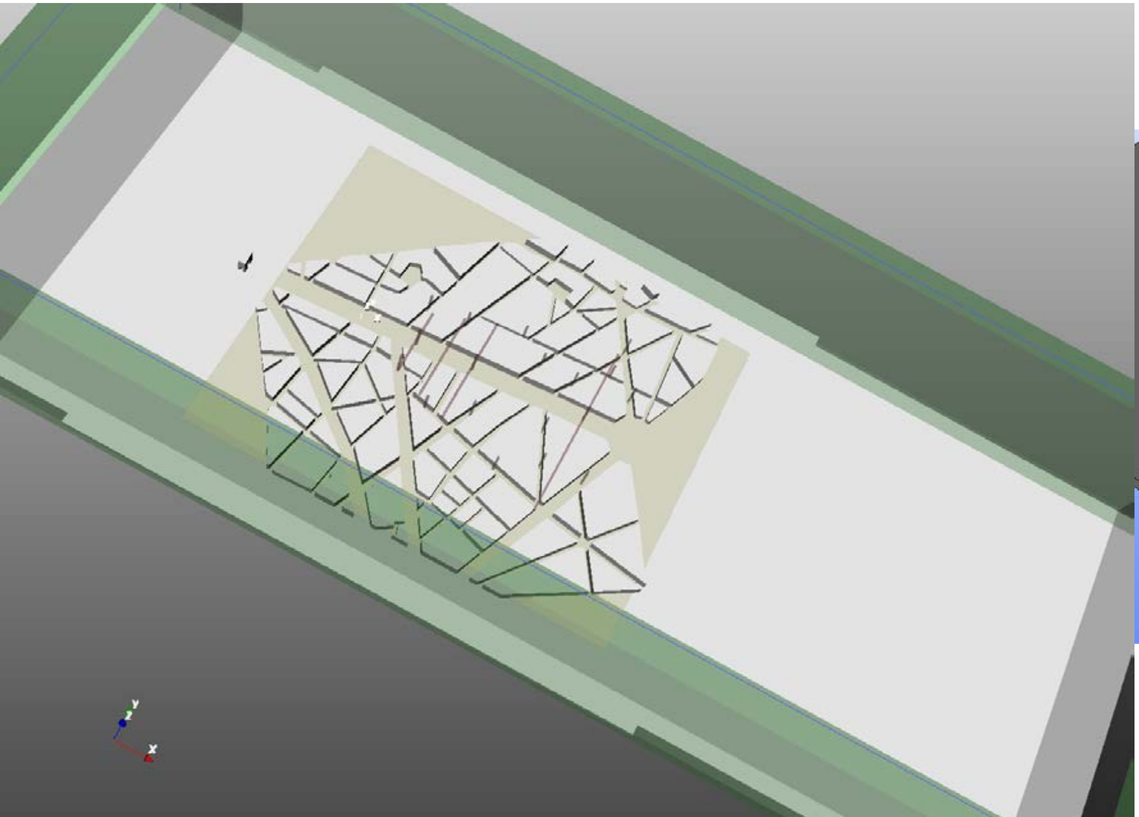


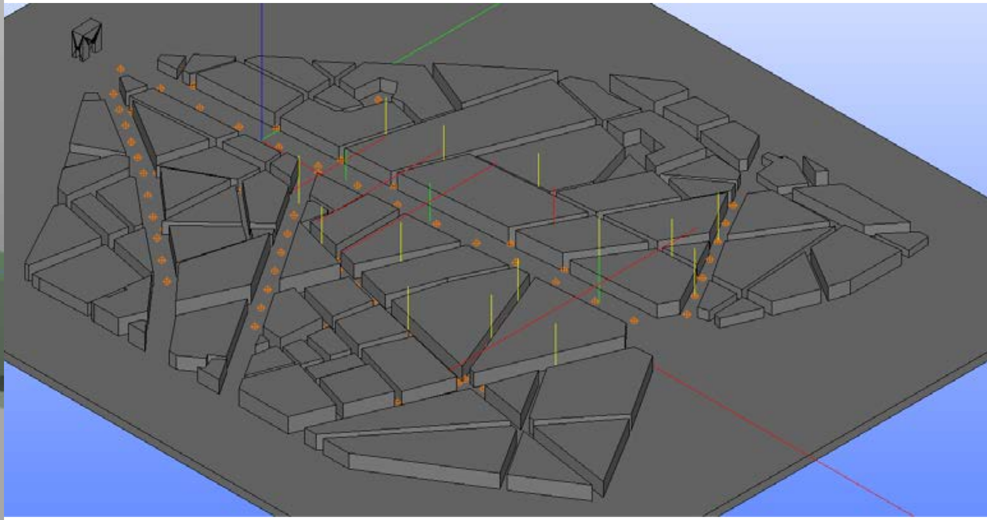
Wind in an helicopter landing area



Wind in an urban area for UAV
Flight mechanics model

Validation: Paris test case
MODITIC Project (European Defence Agency)





Validation: Paris test case
MODITIC Project (European Defence Agency)

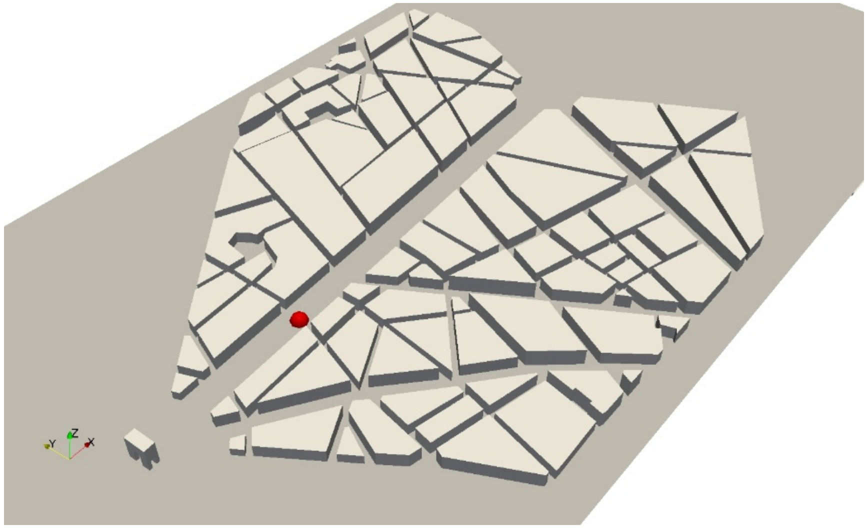
Cas expérimental:

- Échelle : 1/350, Hmoy = 0.074m
- Uref = 1 m/s
- Q= 50 l/min, W0 = 0,076 m/s
- Résultats:
  - Moyenne sur 3mn
  - C traceur en ppm

$$Re \sim 5.10^5$$

Simulations:

- Dx = 0,00175 m
- Dt = 0,00015 s
- Nb Voxels ~ 175.10^6
- Résultats:
  - 10 + 20 s de simulation
  - Moyenne entre 25-30 s



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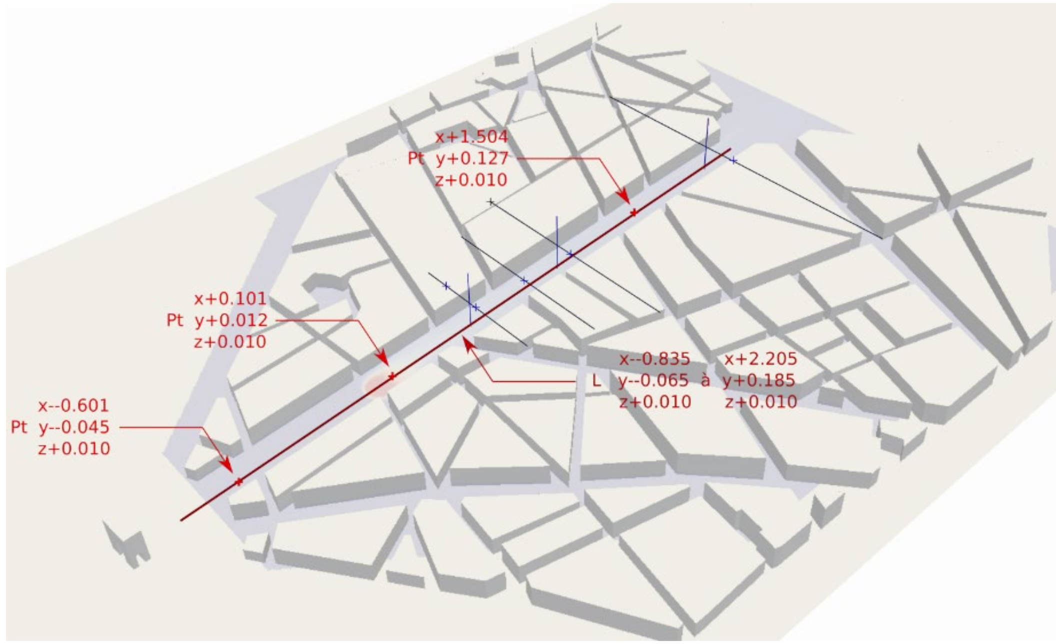

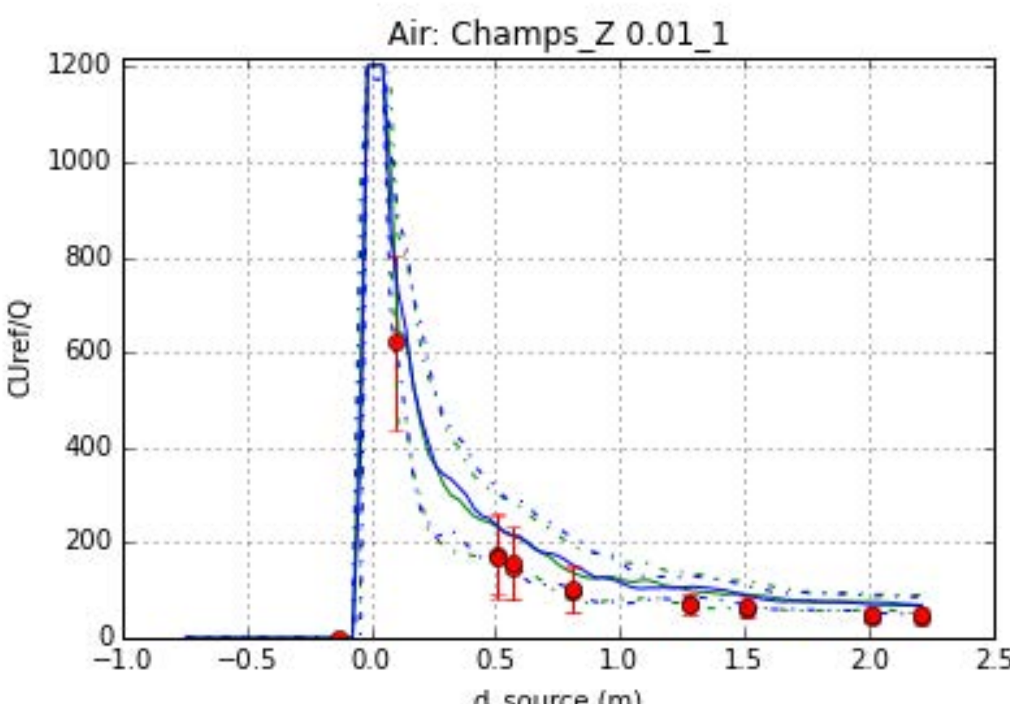

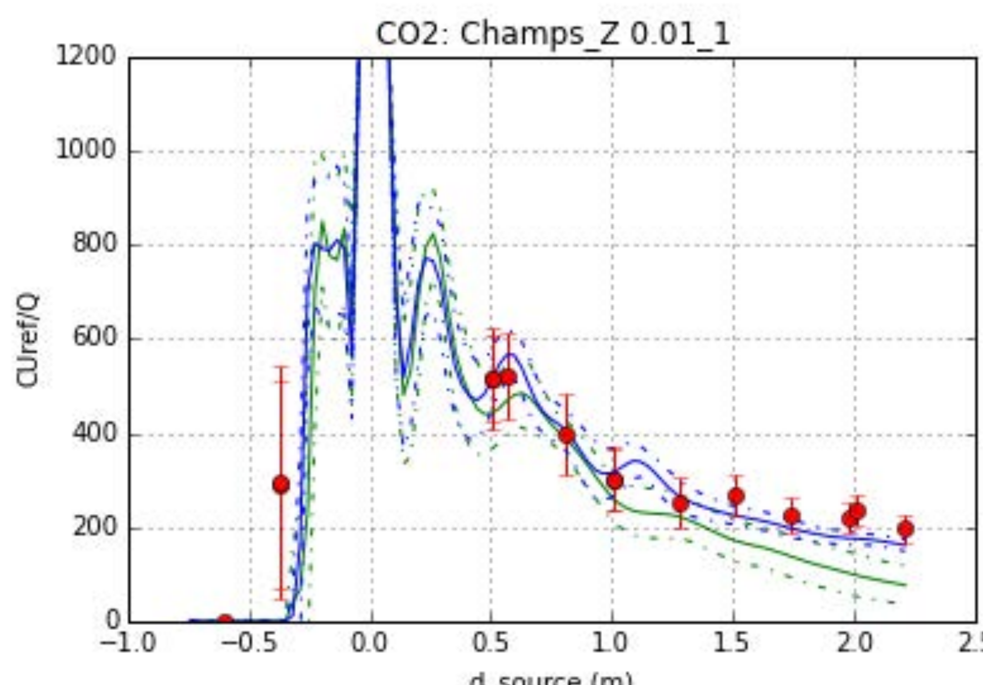

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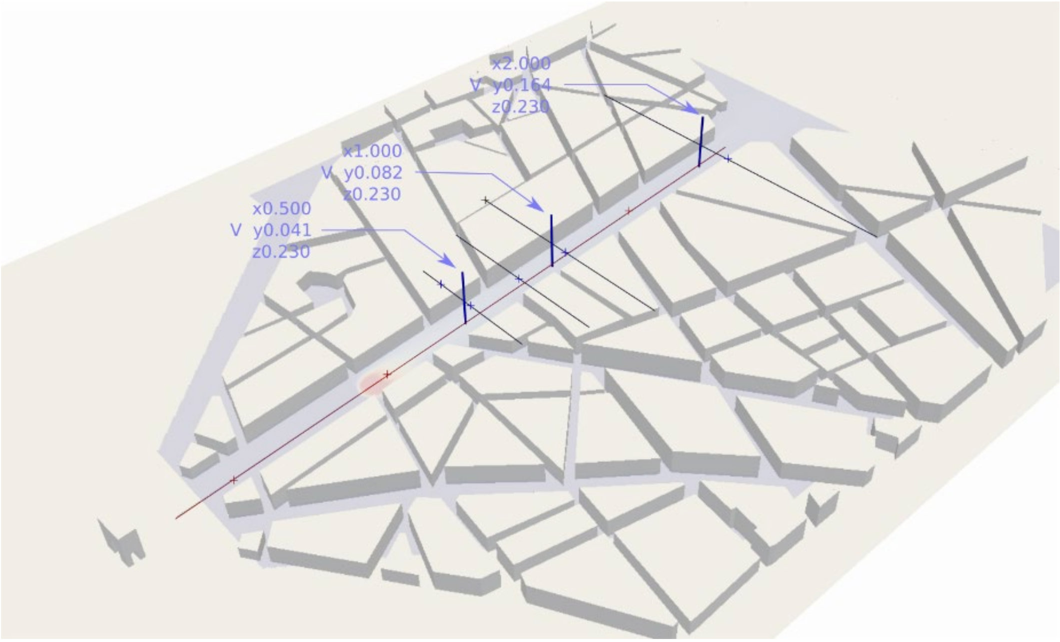

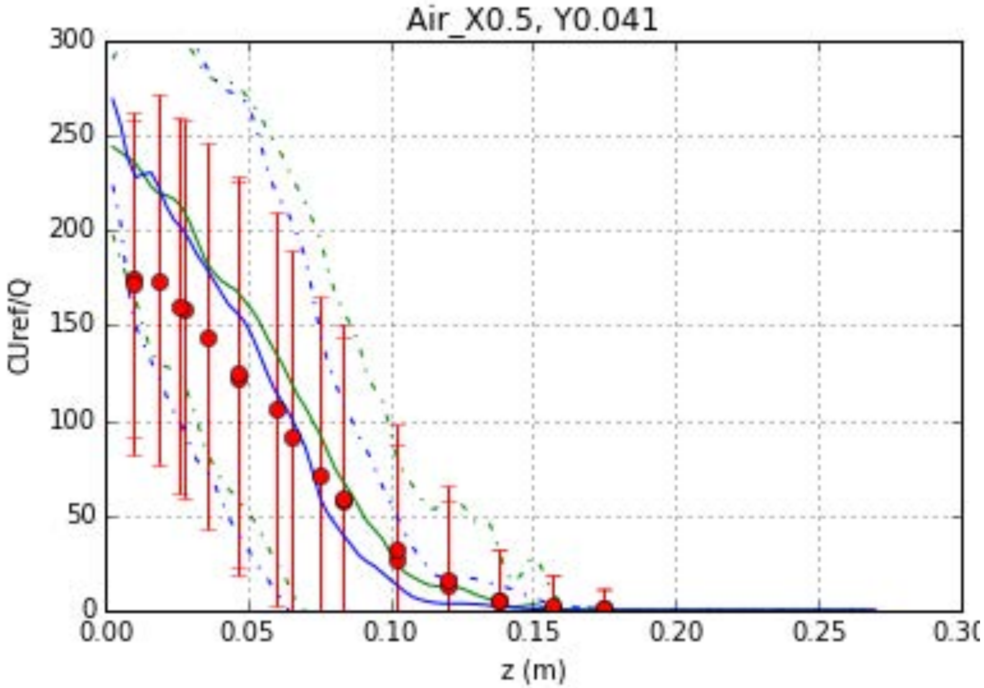

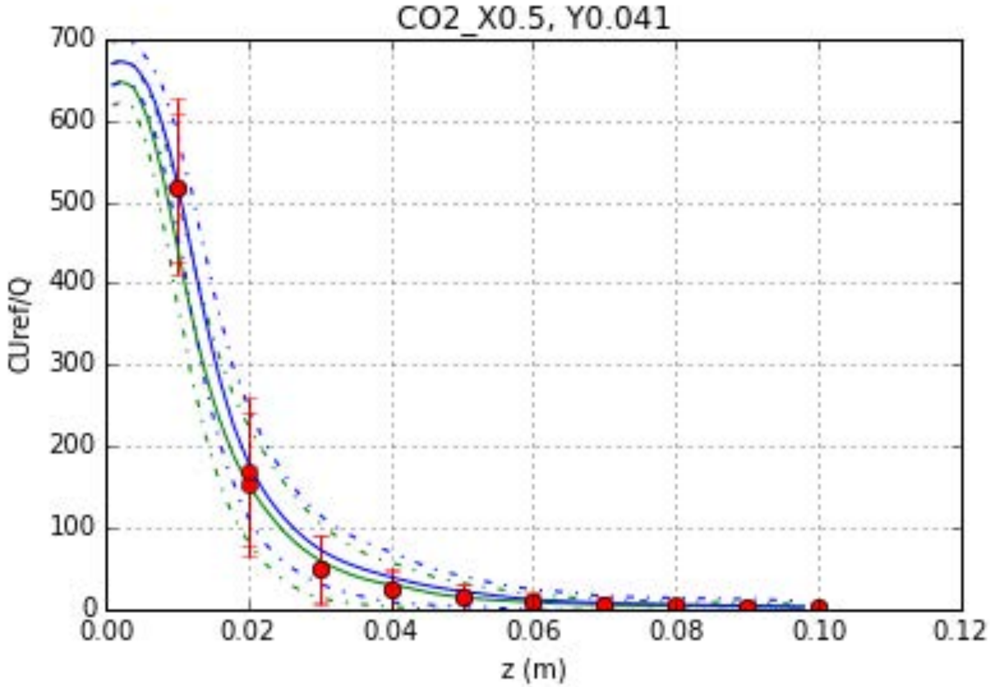

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Thanks!

LAttice-Boltzmann Solver: a new industrial CFD tool

Aix-Marseille University

EC-Lyon

ONERA
CERFACS
LMO
…

ProLB

CS

Renault

Airbus