Turbulence and dissipation in the solar wind

Alain Noullez

with R. Marino, L. Sorriso, V. Carbone, R. Bruno, . . .

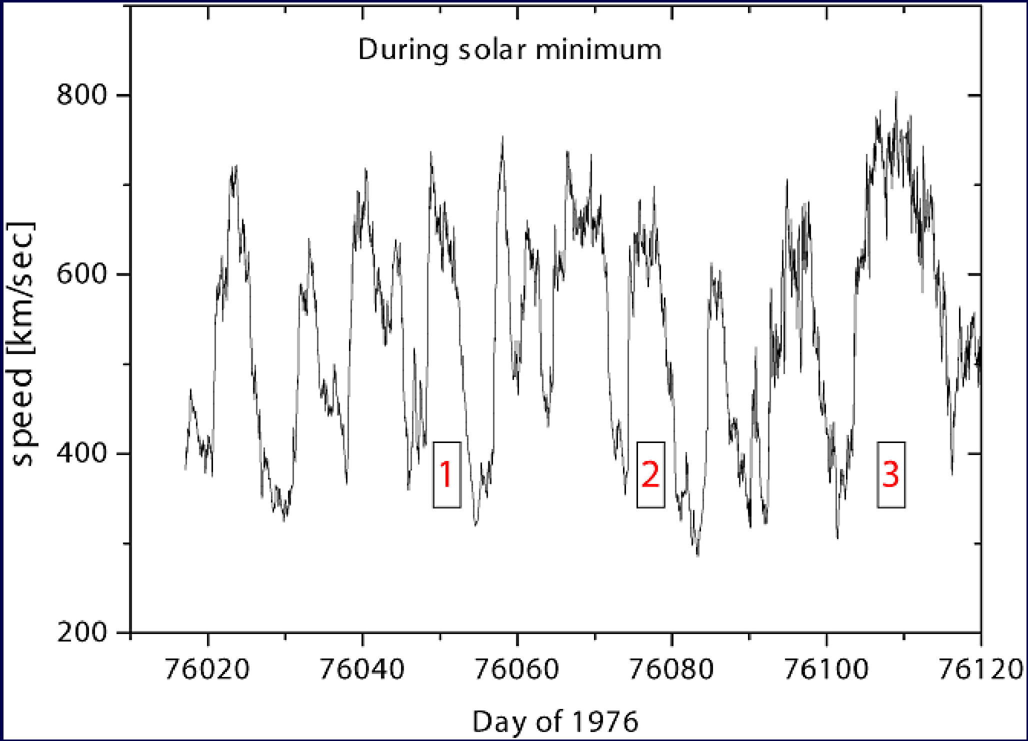
C.N.R.S., U.C.A., Observatoire de la Côte d'Azur, Nice, France

Email : anz@obs-nice.fr

Fluid turbulence Applications in Both Industrial and ENvironmental topics, FAB 60, Marseille, France, July 2019

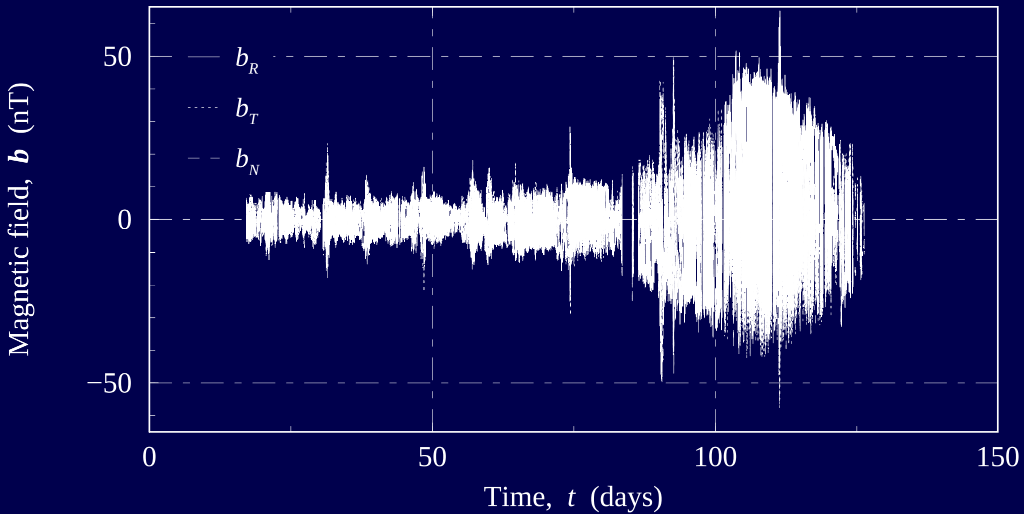
The solar wind

Measurements in the solar wind



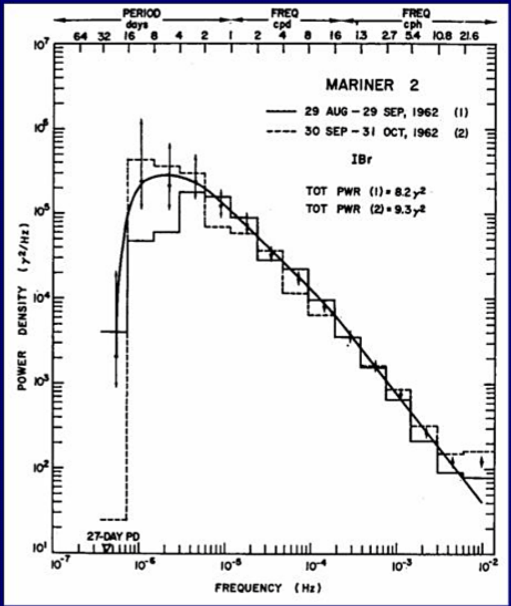
Wind velocity measured by the Helios 2 spacecraft during the year 1976

Magnetic fluctuations in the solar wind

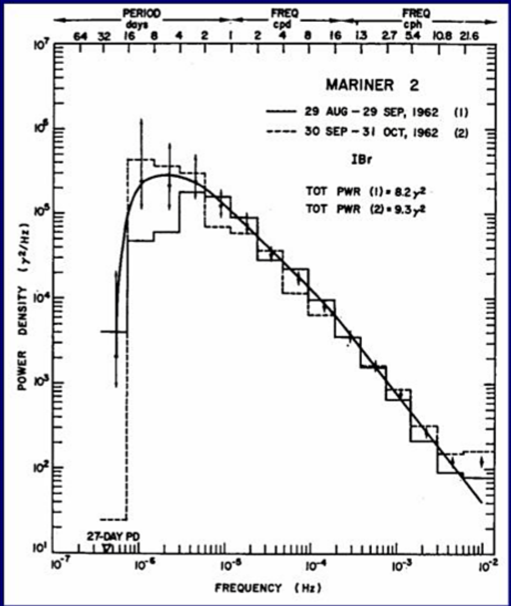


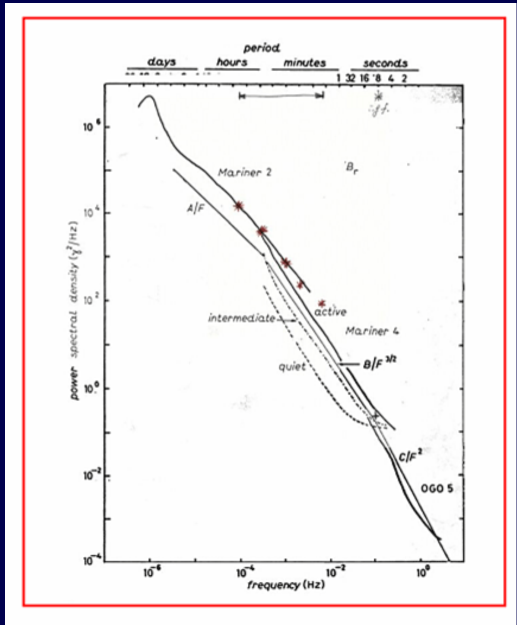
Magnetic field measurements by the Helios 2 spacecraft during the year 1976

Turbulence in the solar wind



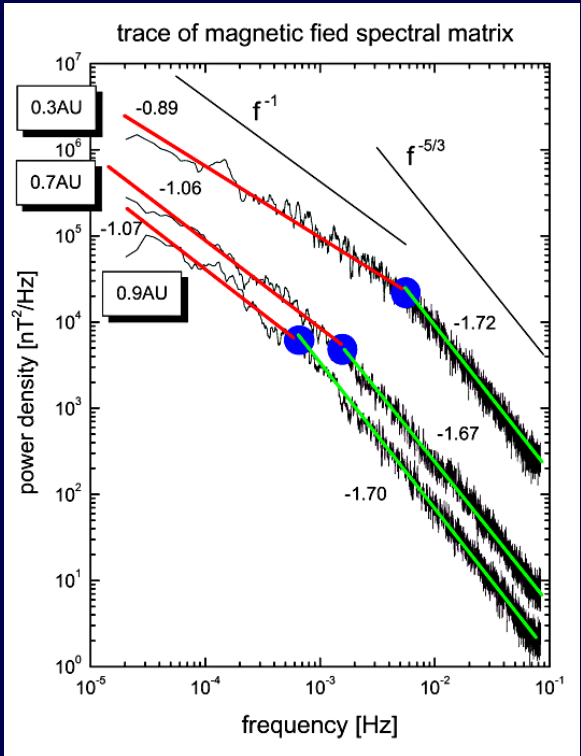
Turbulence in the solar wind





Power spectrum of the magnetic fluctuations

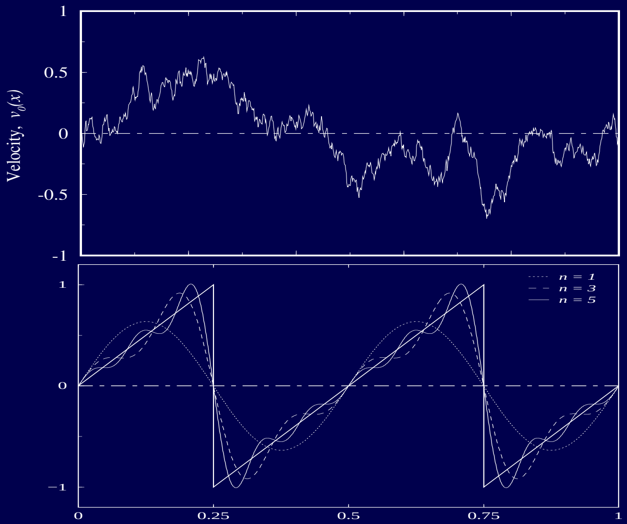
Turbulent spectrum in the solar wind

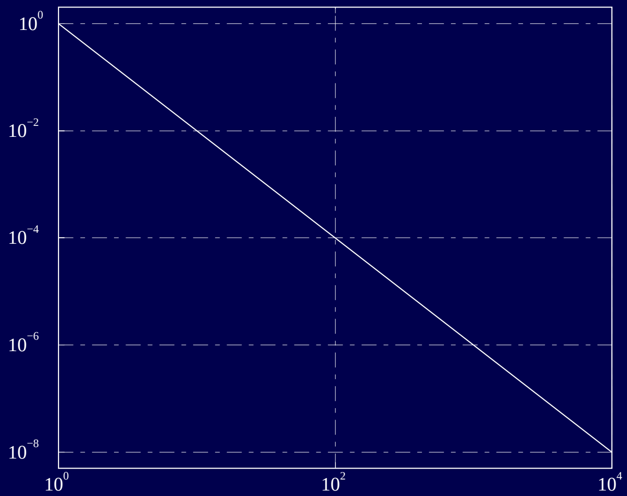


Magnetic power spectra measured by the Helios 2 spacecraft at different distances

Examples of power spectra

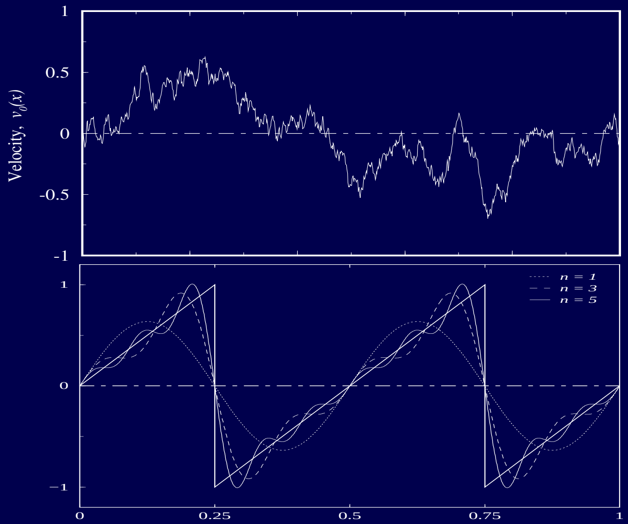
Brownian noise $S_{ww}(\nu) = (2\pi\nu)^{-2} \stackrel{?}{=}$ Sawtooth wave $S_{ss}(\nu) = (2\pi\nu)^{-2}$

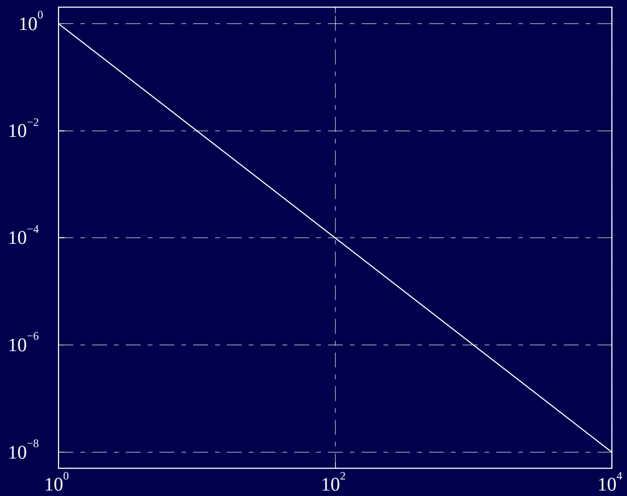




Examples of power spectra

Brownian noise $S_{ww}(\nu) = (2\pi\nu)^{-2} \stackrel{?}{=}$ Sawtooth wave $S_{ss}(\nu) = (2\pi\nu)^{-2}$

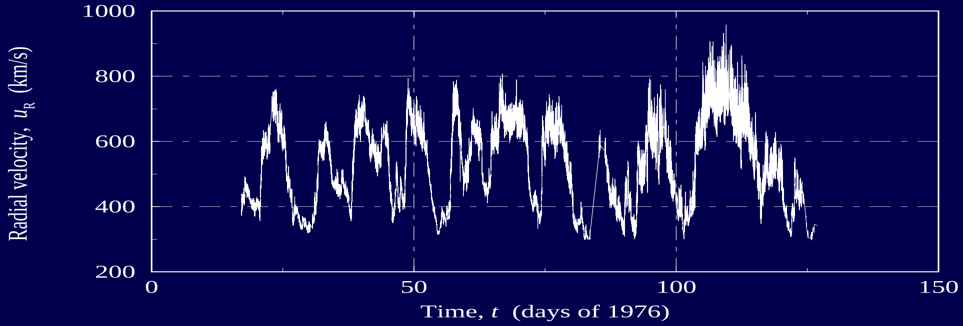




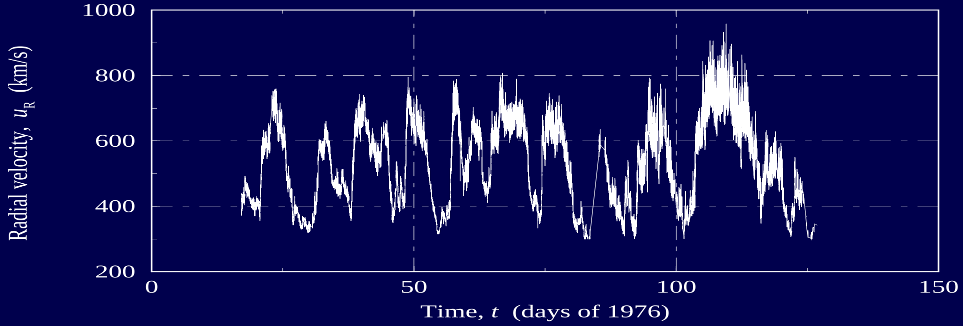
Same spectrum but different type and distribution of singularities

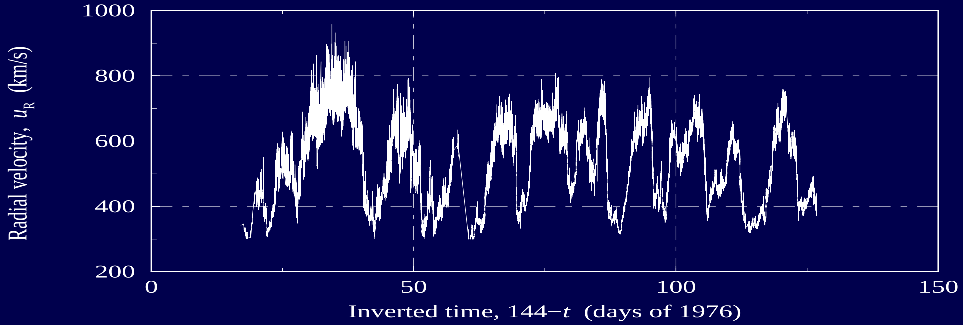
$\Rightarrow$ Necessity to look at other quantities... Higher-order moments ?

Irreversibility of time



Irreversibility of time





Statistics of gradients or increments are not invariant by time-reversal

Energy injection, transfer and dissipation

Ando Hiroshige, The Naruto rapids

Dissipation mechanisms

Consider the velocity difference u_ℓ between two points separated by a distance ℓ. How can it be annihilated ?

Dissipation mechanisms

Consider the velocity difference u_ℓ between two points separated by a distance ℓ. How can it be annihilated ?

- Dissipation by viscosity

$$\partial_t u = \nu \partial^2_{xx} u \quad \Rightarrow \quad \tau_{\mathrm{D}}(\ell) \sim \frac{\ell^2}{\nu}$$

Dissipation mechanisms

Consider the velocity difference u_ℓ between two points separated by a distance ℓ. How can it be annihilated ?

- Dissipation by viscosity

$$\partial_t u = \nu \partial^2_{xx} u \quad \Rightarrow \quad \tau_{\mathrm{D}}(\ell) \sim \frac{\ell^2}{\nu}$$

- Non-linearity

$$\partial_t u + u\,\partial_x u = 0 \quad \Rightarrow \quad \tau_{\mathrm{NL}}(\ell) \sim \frac{\ell}{u_\ell}$$

Dissipation mechanisms

Consider the velocity difference u_ℓ between two points separated by a distance ℓ. How can it be annihilated ?

- Dissipation by viscosity

$$\partial_t u = \nu \partial^2_{xx} u \quad \Rightarrow \quad \tau_{\mathrm{D}}(\ell) \sim \frac{\ell^2}{\nu}$$

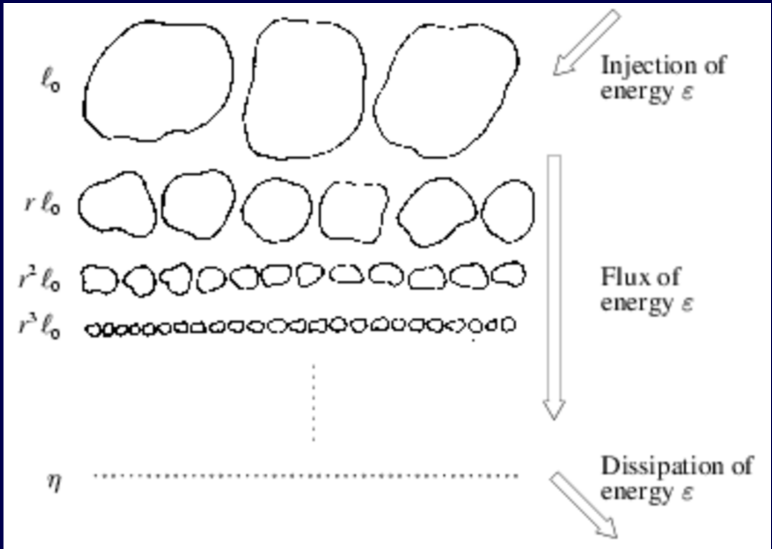
- Non-linearity

$$\partial_t u + u\,\partial_x u = 0 \quad \Rightarrow \quad \tau_{\mathrm{NL}}(\ell) \sim \frac{\ell}{u_\ell}$$

The ratio of these two times at large scale is the Reynolds number

$$\frac{\tau_{\mathrm{D}}(L)}{\tau_{\mathrm{NL}}(L)} = \Re = \frac{UL}{\nu}$$

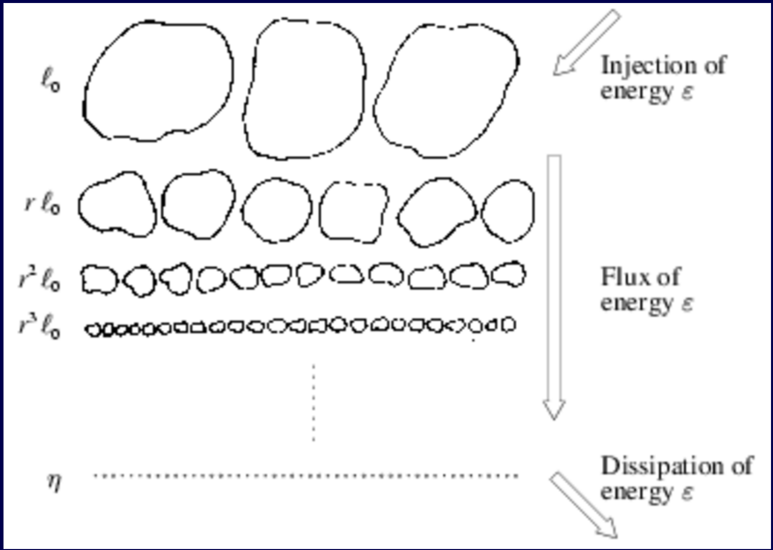
Energy cascade



Injection

$$\varepsilon_D = U^2/\tau_{\mathrm{D}}(L)$$

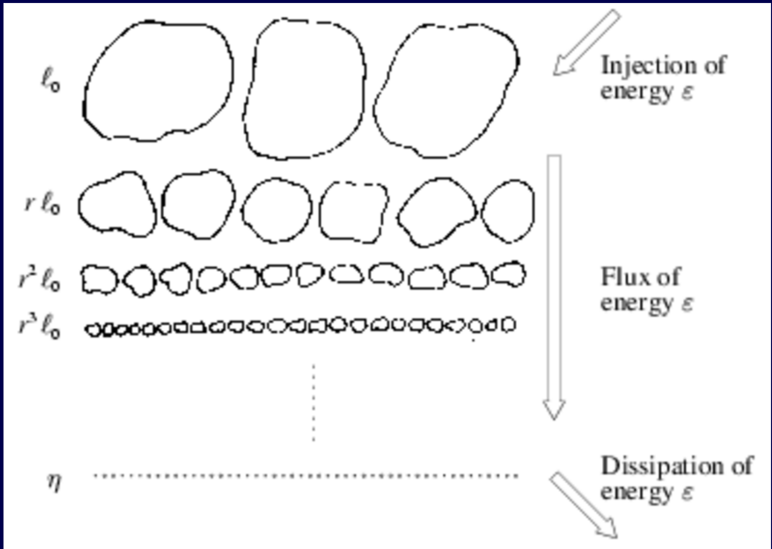
Energy cascade



Injection

$$\varepsilon_D = \nu U^2 / L^2$$

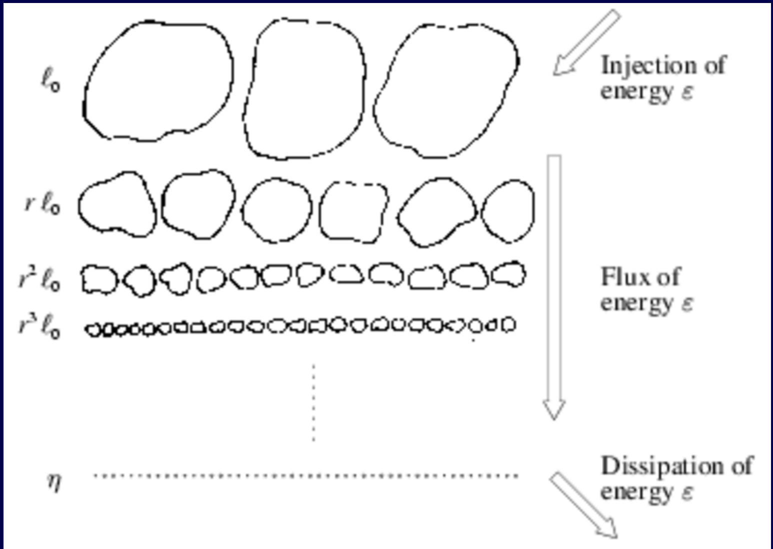
Energy cascade



Injection

$$\varepsilon_L = U^2/\tau_{\mathrm{NL}}(L)$$

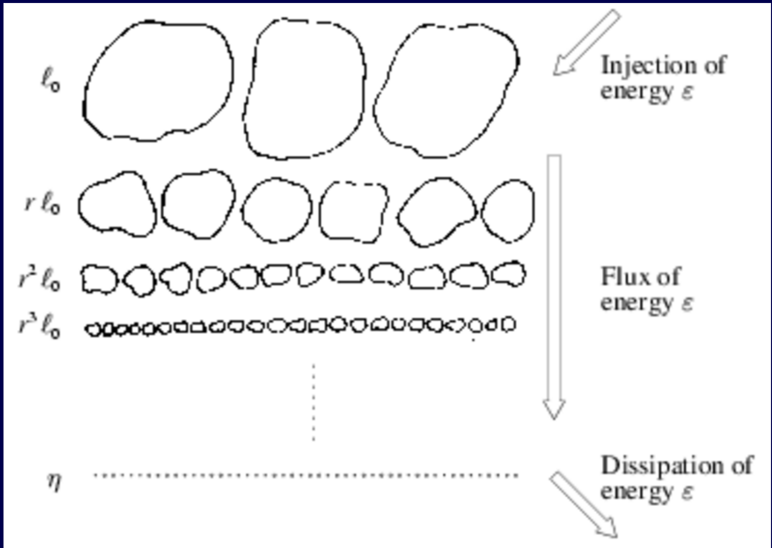
Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

Energy cascade



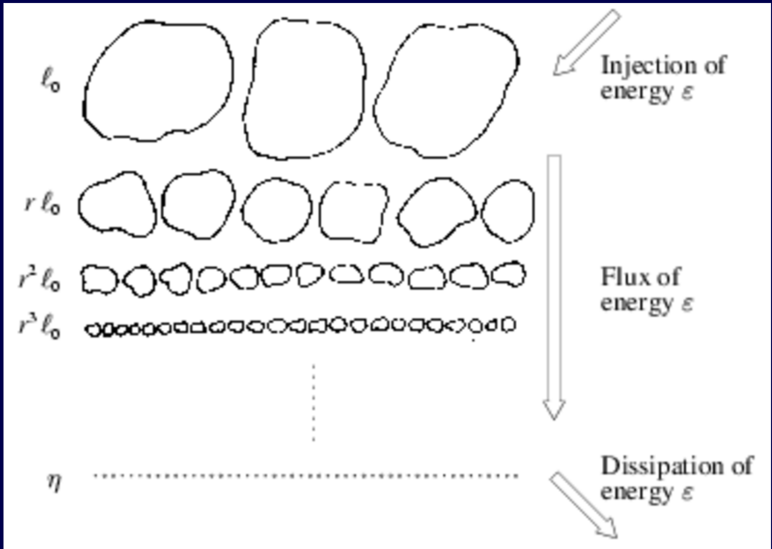
Injection

$$\varepsilon_L = U^3/L$$

Transfer

$$\varepsilon_\ell = u_\ell^3/\ell$$

Energy cascade



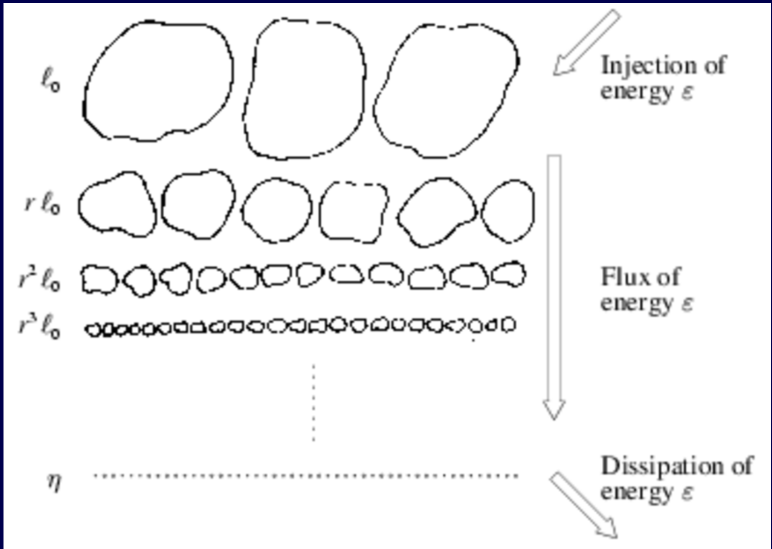
Injection

$$\varepsilon_L = U^3/L$$

Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

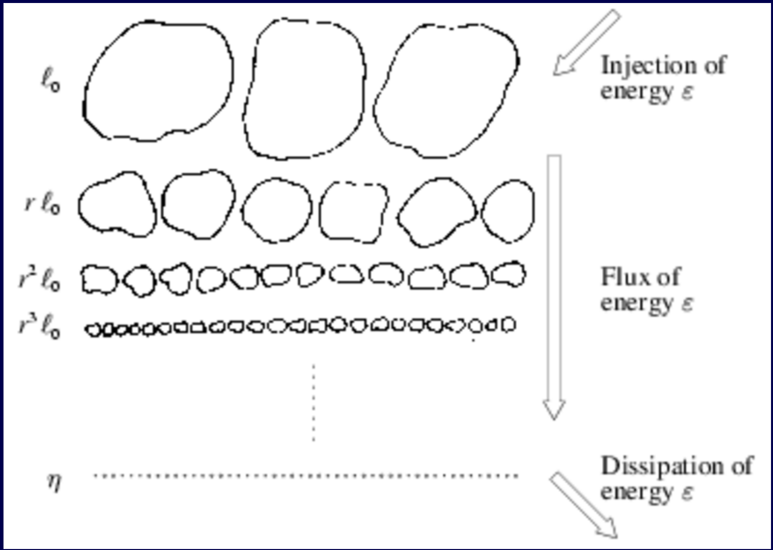
Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Dissipation

$$\varepsilon_\eta = u_\eta^3/\eta$$

Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

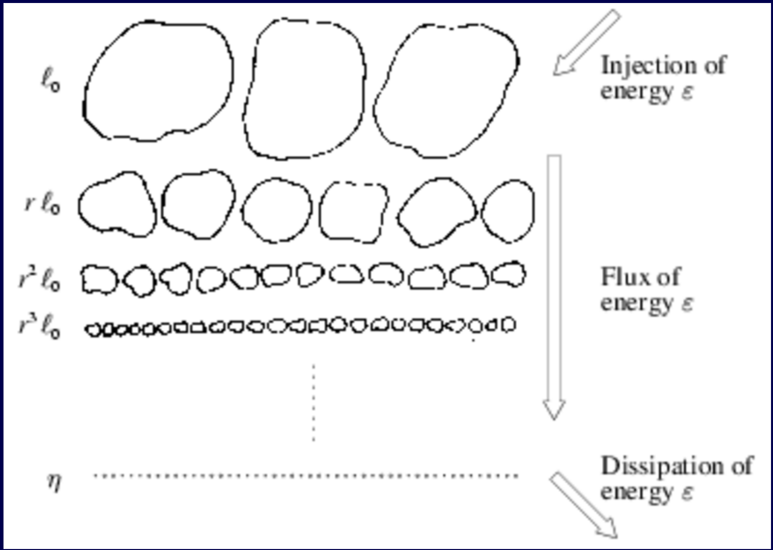
Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Dissipation

$$\varepsilon_\eta = u_\eta^3/\eta = u_\eta^2/\tau_{\mathrm{D}}(\eta)$$

Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

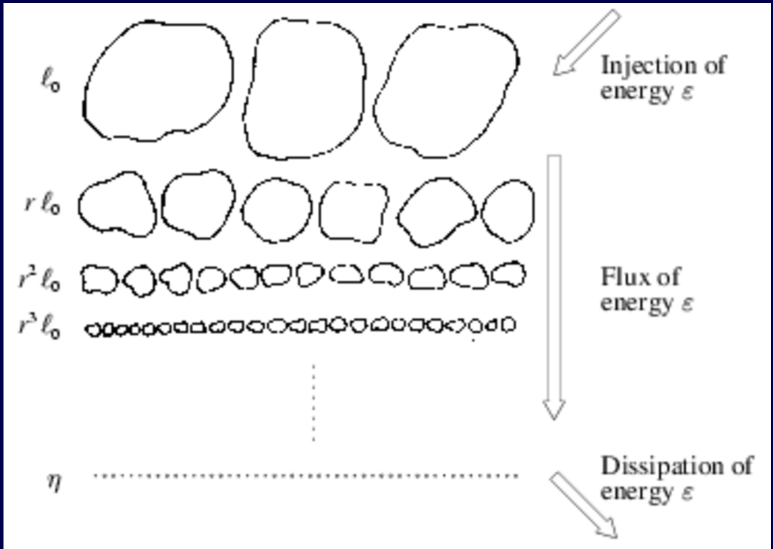
Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Dissipation

$$\varepsilon_\eta = u_\eta^3/\eta = \nu\, u_\eta^2/\eta^2$$

Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

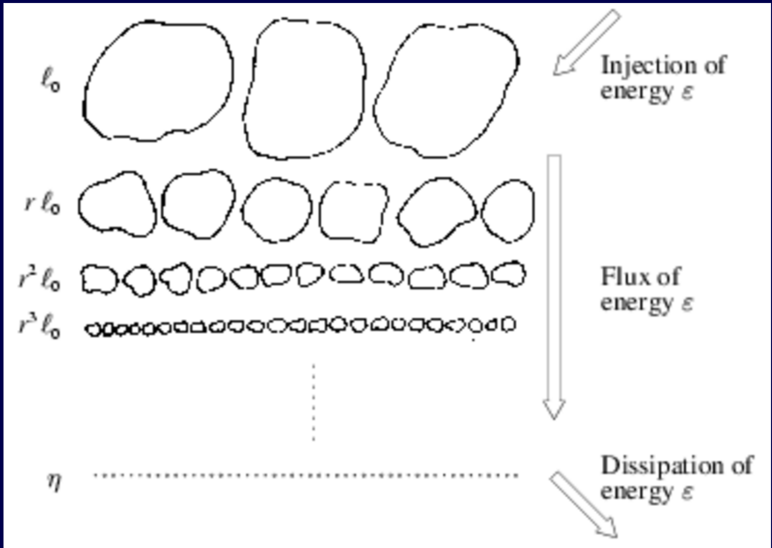
Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Dissipation

$$\varepsilon_\eta = u_\eta^3/\eta = \nu\, u_\eta^2/\eta^2 = \varepsilon_L$$

Energy cascade



Injection

$$\varepsilon_L = U^3/L$$

Transfer

$$\varepsilon_\ell = u_\ell^3/\ell = \varepsilon_L$$

Dissipation

$$\varepsilon_\eta = u_\eta^3/\eta = \nu\, u_\eta^2/\eta^2 = \varepsilon_L$$

Cascade down to the Kolmogorov scale

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \qquad u_\eta = (\nu\varepsilon)^{1/4}, \qquad \Re_\eta = \left(\frac{u_\eta\,\eta}{\nu}\right) = 1$$

Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla}_x \boldsymbol{u} = \frac{-1}{\rho} \boldsymbol{\nabla}_x P + \nu \nabla_x^2 \boldsymbol{u} + \boldsymbol{f}$$

at two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$ to get an equation for the velocity difference $\Delta \boldsymbol{u}(\boldsymbol{r}; \boldsymbol{x}, t) \equiv \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}, t) - \boldsymbol{u}(\boldsymbol{x}, t)$

Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} \boldsymbol{u} = \frac{-1}{\rho} \boldsymbol{\nabla}_{\boldsymbol{x}} P + \nu \nabla^2_{\boldsymbol{x}} \boldsymbol{u} + \boldsymbol{f}$$

at two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$ to get an equation
for the velocity difference $\Delta \boldsymbol{u}(\boldsymbol{r}; \boldsymbol{x}, t) \equiv \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}, t) - \boldsymbol{u}(\boldsymbol{x}, t)$
then multiply by $2\,\Delta \boldsymbol{u}$ and average over the space $\boldsymbol{x}$ to get

$$\partial_t \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle + \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = 2\,\nu \nabla^2_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle - 4\,\nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla}_x \boldsymbol{u} = \frac{-1}{\rho} \boldsymbol{\nabla}_x P + \nu \nabla^2_{\boldsymbol{x}} \boldsymbol{u} + \boldsymbol{f}$$

at two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$ to get an equation
for the velocity difference $\Delta \boldsymbol{u}(\boldsymbol{r}; \boldsymbol{x}, t) \equiv \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}, t) - \boldsymbol{u}(\boldsymbol{x}, t)$
then multiply by $2\,\Delta \boldsymbol{u}$ and average over the space $\boldsymbol{x}$ to get

$$\partial_t \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle + \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = 2\,\nu\, \nabla^2_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle - 4\,\nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

so that in stationary conditions and in the limit $\nu \longrightarrow 0$

$$\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = -4\,\varepsilon \qquad \varepsilon \equiv -\frac{\mathrm{d}E}{\mathrm{d}t} = \nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} \boldsymbol{u} = \frac{-1}{\rho} \boldsymbol{\nabla}_{\boldsymbol{x}} P + \nu \nabla^2_{\boldsymbol{x}} \boldsymbol{u} + \boldsymbol{f}$$

at two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$ to get an equation
for the velocity difference $\Delta \boldsymbol{u}(\boldsymbol{r}; \boldsymbol{x}, t) \equiv \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}, t) - \boldsymbol{u}(\boldsymbol{x}, t)$
then multiply by $2\,\Delta \boldsymbol{u}$ and average over the space $\boldsymbol{x}$ to get

$$\partial_t \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle + \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = 2\,\nu\, \nabla^2_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle - 4\,\nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

so that in stationary conditions and in the limit $\nu \to 0$

$$\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = -4\,\varepsilon \qquad \varepsilon \equiv -\frac{\mathrm{d}E}{\mathrm{d}t} = \nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

which in case of isotropy implies $\left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = -4/3\,\varepsilon\, \boldsymbol{r}$ (Yaglom)

Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} \boldsymbol{u} = \frac{-1}{\rho} \boldsymbol{\nabla}_{\boldsymbol{x}} P + \nu \nabla^2_{\boldsymbol{x}} \boldsymbol{u} + \boldsymbol{f}$$

at two points $\boldsymbol{x}$ and $\boldsymbol{x}' \equiv \boldsymbol{x} + \boldsymbol{r}$ to get an equation
for the velocity difference $\Delta \boldsymbol{u}(\boldsymbol{r}; \boldsymbol{x}, t) \equiv \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}, t) - \boldsymbol{u}(\boldsymbol{x}, t)$
then multiply by $2\,\Delta \boldsymbol{u}$ and average over the space $\boldsymbol{x}$ to get

$$\partial_t \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle + \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = 2\,\nu\, \nabla^2_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle - 4\,\nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

so that in stationary conditions and in the limit $\nu \to 0$

$$\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = -4\,\varepsilon \qquad \varepsilon \equiv -\frac{\mathrm{d}E}{\mathrm{d}t} = \nu \left\langle |\boldsymbol{\nabla} \boldsymbol{u}|^2 \right\rangle$$

which in case of isotropy implies $\left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle = -4/3\,\varepsilon\, \boldsymbol{r}$ (Yaglom)
and using local isotropy again $\left\langle [\Delta u_i]^3 \right\rangle = -4/5\,\varepsilon\, r_i$ (Kolmogorov)

"Exact" conservation laws

$$\left.\begin{array}{c}\dfrac{12}{d(d+2)}\,\varepsilon\, r_i \\ \dfrac{4}{d}\,\varepsilon\,\boldsymbol{r} \\ 4\,\varepsilon\end{array}\right\} = \left\{\begin{array}{c} -\left\langle [\Delta u_i]^3 \right\rangle \\ -\left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle \\ -\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \left\langle |\Delta \boldsymbol{u}|^2 \Delta \boldsymbol{u} \right\rangle \end{array}\right. + \left\{\begin{array}{lr} 2\,d\,\nu\,\partial_i \left\langle [\Delta u_i]^2 \right\rangle & (1c\,1d) \\ 2\,\nu\,\boldsymbol{\nabla}_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle & (3c\,1d) \\ 2\,\nu\,\nabla^2_{\boldsymbol{r}} \left\langle |\Delta \boldsymbol{u}|^2 \right\rangle & (3c\,3d) \end{array}\right.$$

Total dissipation at scale $\boldsymbol{r}$	Transfer to scales $< \boldsymbol{r}$	Viscous dissipation at scale $\boldsymbol{r}$	Exp

- Scale by scale energy conservation $\Rightarrow$ no pileup of energy
- Increasing generality (Monin's law valid for anisotropic flows)
- Increasing experimental difficulty

The Kolmogorov spectrum

In the inertial range

$L \ll \ell \ll \eta$

$$\langle (\Delta u_{\parallel}(\ell))^3 \rangle = -4/5\, \varepsilon\, \ell$$

The Kolmogorov spectrum

In the inertial range

$L \ll \ell \ll \eta$

$$\langle (\Delta u_{\parallel}(\ell))^3 \rangle = -4/5\, \varepsilon\, \ell$$

By self-similarity

$$\langle (\Delta u_{\parallel}(\ell))^p \rangle = C_p\, (\varepsilon\, \ell)^{p/3}$$
$$\langle (\Delta u_{\parallel}(\ell))^2 \rangle = C_2\, (\varepsilon\, \ell)^{2/3}$$

The Kolmogorov spectrum

In the inertial range

$L \ll \ell \ll \eta$

$$\langle (\Delta u_{\|}(\ell))^3 \rangle = -4/5\, \varepsilon\, \ell$$

By self-similarity

$$\langle (\Delta u_{\|}(\ell))^p \rangle = C_p\, (\varepsilon\, \ell)^{p/3}$$

$$\langle (\Delta u_{\|}(\ell))^2 \rangle = C_2\, (\varepsilon\, \ell)^{2/3}$$

$$E(k) = C_{\mathrm{K}}\, \varepsilon^{2/3}\, k^{-5/3}$$

The Kolmogorov spectrum

In the inertial range

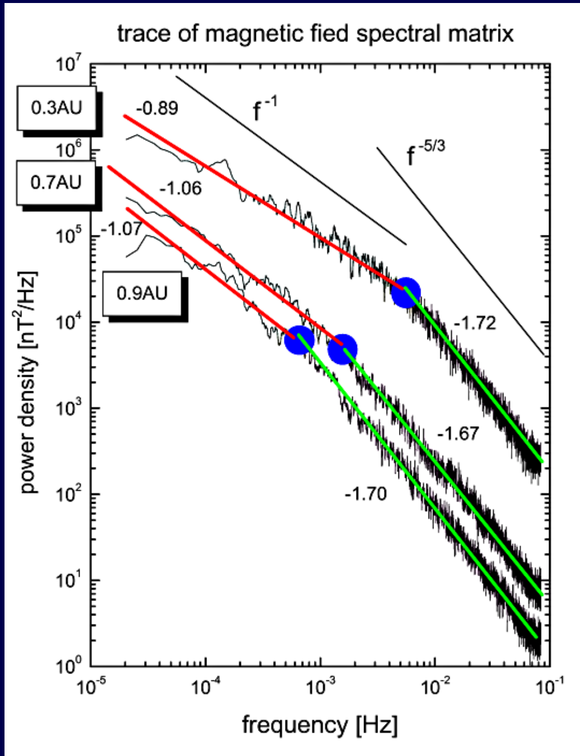
$L \ll \ell \ll \eta$

$$\langle (\Delta u_{\|}(\ell))^3 \rangle = -4/5\, \varepsilon\, \ell$$

By self-similarity

$$\langle (\Delta u_{\|}(\ell))^p \rangle = C_p\, (\varepsilon\, \ell)^{p/3}$$
$$\langle (\Delta u_{\|}(\ell))^2 \rangle = C_2\, (\varepsilon\, \ell)^{2/3}$$

$$E(k) = C_{\rm K}\, \varepsilon^{2/3}\, k^{-5/3}$$



Magneto-Hydrodynamics

For a charged fluid at velocities $u \ll c$ in a mean magnetic field $\boldsymbol{B}_0$ velocity $\boldsymbol{u}$ and magnetic field $\boldsymbol{b}$ perturbations obey the MHD equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{\nabla} \cdot \boldsymbol{b} = 0 \qquad \boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \frac{1}{4\pi\rho} (\boldsymbol{\nabla} \times \boldsymbol{b}) \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{u}$$

$$\partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{b}$$

Magneto-Hydrodynamics

For a charged fluid at velocities $u \ll c$ in a mean magnetic field $\boldsymbol{B}_0$ velocity $\boldsymbol{u}$ and magnetic field $\boldsymbol{b}$ perturbations obey the MHD equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{\nabla} \cdot \boldsymbol{b} = 0 \qquad \boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \frac{1}{4\pi\rho}(\boldsymbol{\nabla} \times \boldsymbol{b}) \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{u}$$

$$\partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{b}$$

Consider the Elsässer variables $\boldsymbol{z}^{\pm} = \boldsymbol{u} \pm (4\pi\rho)^{-1/2}\,\boldsymbol{b}$

$$\boldsymbol{\nabla} \cdot \boldsymbol{z}^{\pm} = 0$$

$$\partial_t \boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} = -\boldsymbol{\nabla} p^{\star} + \boldsymbol{c}_{\mathrm{A}} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} + \nu^{\pm} \nabla^2 \boldsymbol{z}^{\pm} + \nu^{\mp} \nabla^2 \boldsymbol{z}^{\mp}$$

where $\boldsymbol{c}_{\mathrm{A}}$ is the Alfvén velocity $(4\pi\rho)^{-1/2}\boldsymbol{B}_0$ and $\nu^{\pm} = (\nu \pm \eta)/2$

Yaglom equations for MHD turbulence

Derivation as for the Navier-Stokes equations, excepted that one of the Elsässer variables $\boldsymbol{z}^{\pm}$ is transported by the other $\boldsymbol{z}^{\mp}$

$$Y^{\pm}(\ell) \equiv \left\langle |\Delta \boldsymbol{z}^{\pm}|^{2} \Delta z_{\parallel}^{\mp} \right\rangle = -\frac{4}{3}\, \varepsilon^{\pm}\, \ell$$

Yaglom equations for MHD turbulence

Derivation as for the Navier-Stokes equations,
excepted that one of the Elsässer variables $\boldsymbol{z}^{\pm}$
is transported by the other $\boldsymbol{z}^{\mp}$

$$Y^{\pm}(\ell) \equiv \left\langle |\Delta \boldsymbol{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp} \right\rangle = -\frac{4}{3}\,\varepsilon^{\pm}\,\ell$$

which in terms of velocity $\boldsymbol{u}$ and magnetic field $\boldsymbol{b}$ reads

$$\left\langle \left[|\Delta \boldsymbol{u}|^2 + |\Delta \boldsymbol{b}|^2 \pm 2\,\Delta \boldsymbol{u} \cdot \Delta \boldsymbol{b} \right] \left(\Delta u_{\parallel} \mp \Delta b_{\parallel} \right) \right\rangle = -\frac{4}{3}\,\varepsilon^{\pm}\,\ell$$

coupling energy $|\boldsymbol{u}|^2 + |\boldsymbol{b}|^2$ and cross-helicity $\boldsymbol{u} \cdot \boldsymbol{b}$ cascades

Yaglom equations for MHD turbulence

Derivation as for the Navier-Stokes equations,
excepted that one of the Elsässer variables $\boldsymbol{z}^{\pm}$
is transported by the other $\boldsymbol{z}^{\mp}$

$$Y^{\pm}(\ell) \equiv \left\langle |\Delta \boldsymbol{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp} \right\rangle = -\frac{4}{3}\,\varepsilon^{\pm}\,\ell$$

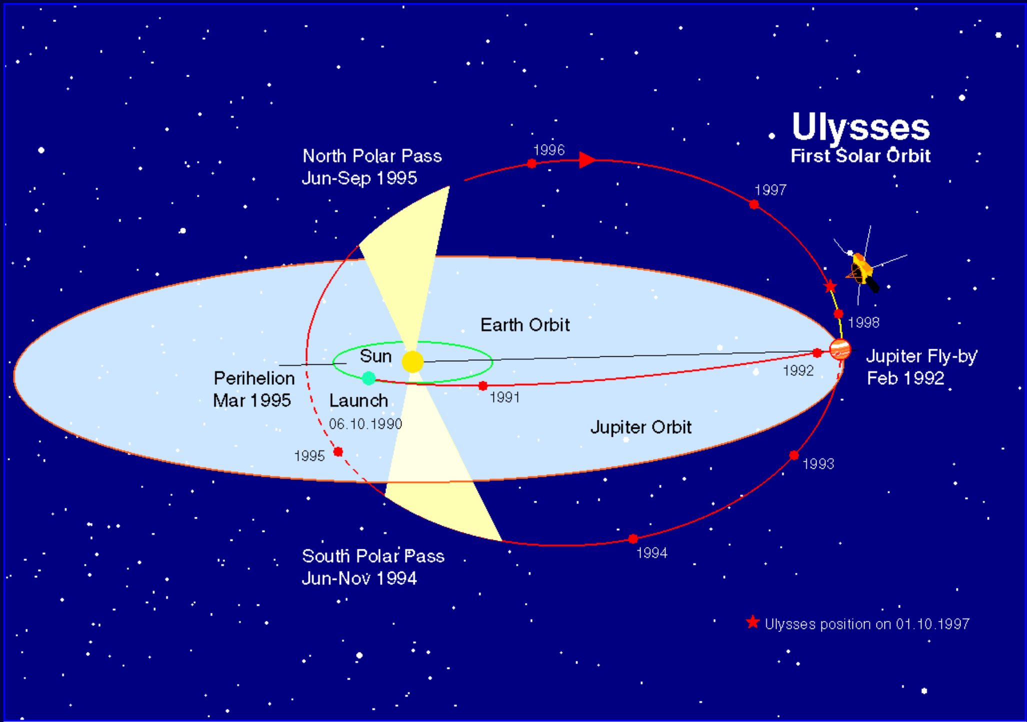
which in terms of velocity $\boldsymbol{u}$ and magnetic field $\boldsymbol{b}$ reads

$$\left\langle \left[|\Delta \boldsymbol{u}|^2 + |\Delta \boldsymbol{b}|^2 \pm 2\,\Delta \boldsymbol{u}\cdot\Delta \boldsymbol{b}\right] \left(\Delta u_{\parallel} \mp \Delta b_{\parallel}\right)\right\rangle = -\frac{4}{3}\,\varepsilon^{\pm}\,\ell$$

coupling energy $|\boldsymbol{u}|^2 + |\boldsymbol{b}|^2$ and cross-helicity $\boldsymbol{u}\cdot\boldsymbol{b}$ cascades

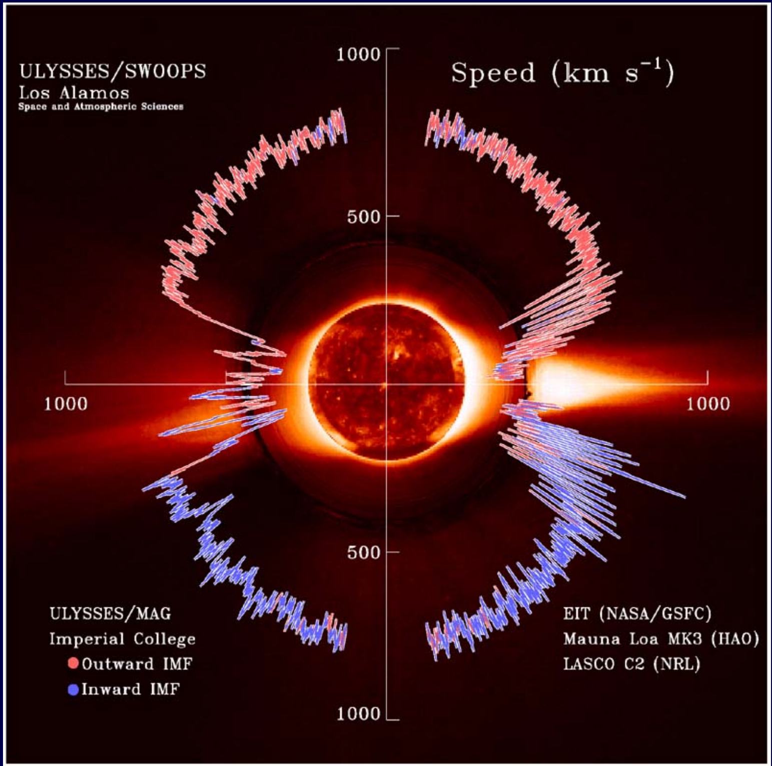
Does it hold in the solar wind ???

Verification in the Ulysses data



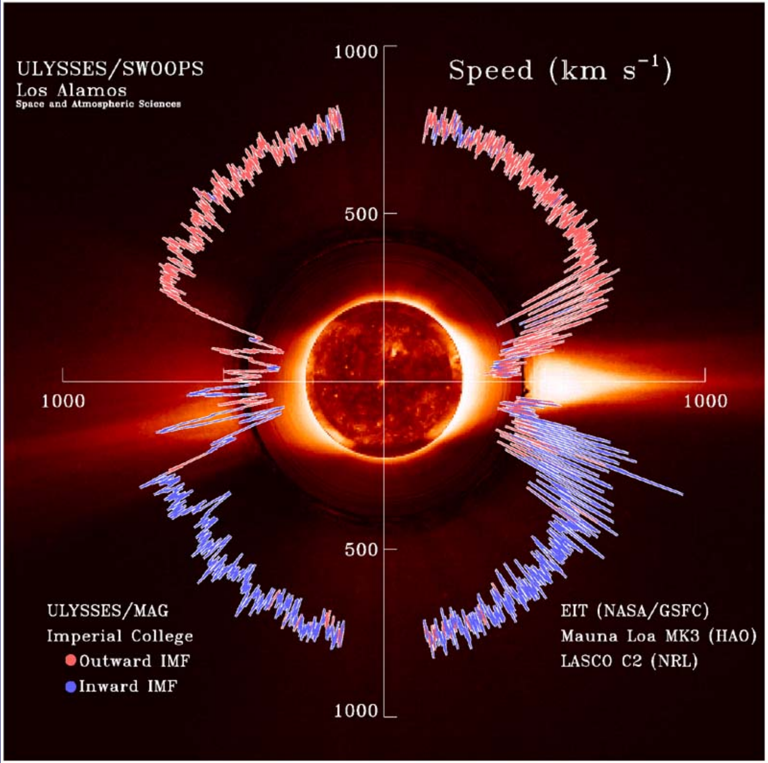
The Ulysses mission first north polar pass during the year 1996 at solar minimum

The solar wind as seen by Ulysses



High-latitude $\theta > 35\,^\circ$
fast $|u| > 700\,\mathrm{km/s}$
polar solar wind
near solar minimum
1995 – 1996

The solar wind as seen by Ulysses



High-latitude $\theta > 35\,^\circ$
fast $|u| > 700\,\mathrm{km/s}$
polar solar wind
near solar minimum
1995 – 1996

Use 8 - min average data
of $\rho, \boldsymbol{u}$ and $\boldsymbol{b}$
to build the Elsässer
variables $\boldsymbol{z}^{\pm}$

Data processing

- Reconstruction of the spatial (radial) dependence

using the Taylor's frozen-flow method

$$
\begin{aligned}
\boldsymbol{u}(\boldsymbol{x}, t+\tau) &\approx \boldsymbol{u}(\boldsymbol{x} - \overline{\boldsymbol{u}}\,\tau, t) \\
\boldsymbol{z}^{\pm}(\boldsymbol{x}, t+\tau) - \boldsymbol{z}^{\pm}(\boldsymbol{x}, t) &\approx \boldsymbol{z}^{\pm}(\boldsymbol{x} - \overline{u_r}\,\tau\,\mathbf{1}_R, t) - \boldsymbol{z}^{\pm}(\boldsymbol{x}, t) \\
&\approx \Delta \boldsymbol{z}^{\pm}(-\overline{u_r}\,\tau\,\mathbf{1}_R, t)
\end{aligned}
$$

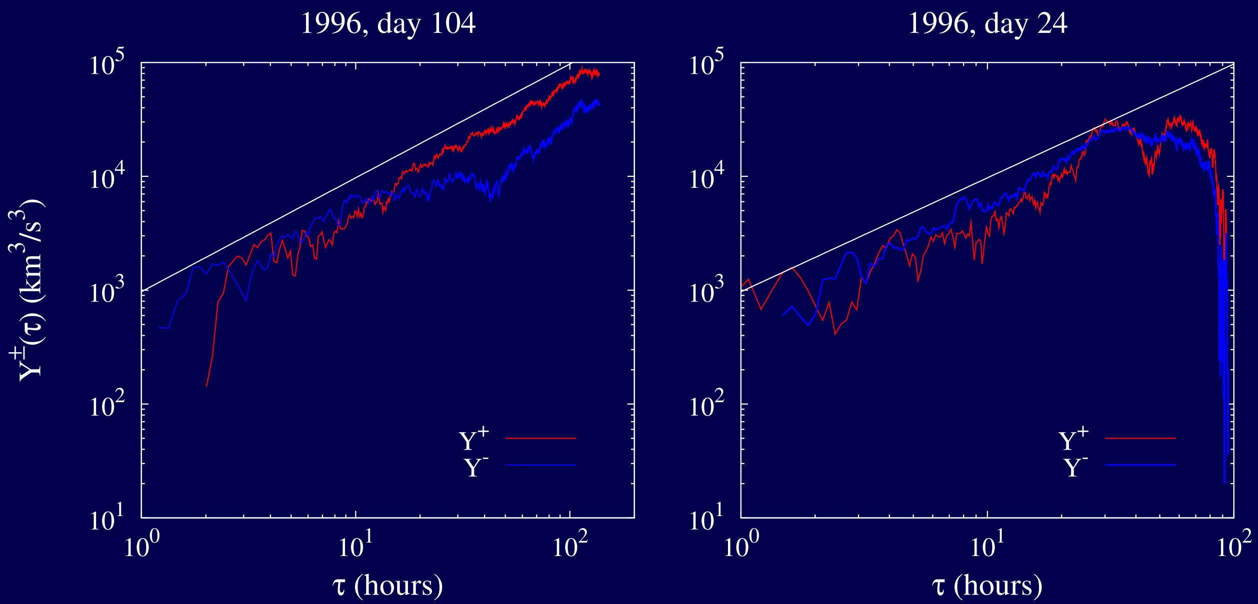
Data processing

• Reconstruction of the spatial (radial) dependence using the Taylor's frozen-flow method

$$
\begin{aligned}
\boldsymbol{u}(\boldsymbol{x}, t+\tau) &\approx \boldsymbol{u}(\boldsymbol{x} - \overline{\boldsymbol{u}}\,\tau, t) \\
\boldsymbol{z}^{\pm}(\boldsymbol{x}, t+\tau) - \boldsymbol{z}^{\pm}(\boldsymbol{x}, t) &\approx \boldsymbol{z}^{\pm}(\boldsymbol{x} - \overline{u_r}\,\tau\,\mathbf{1}_R, t) - \boldsymbol{z}^{\pm}(\boldsymbol{x}, t) \\
&\approx \Delta\boldsymbol{z}^{\pm}(-\overline{u_r}\,\tau\,\mathbf{1}_R, t)
\end{aligned}
$$

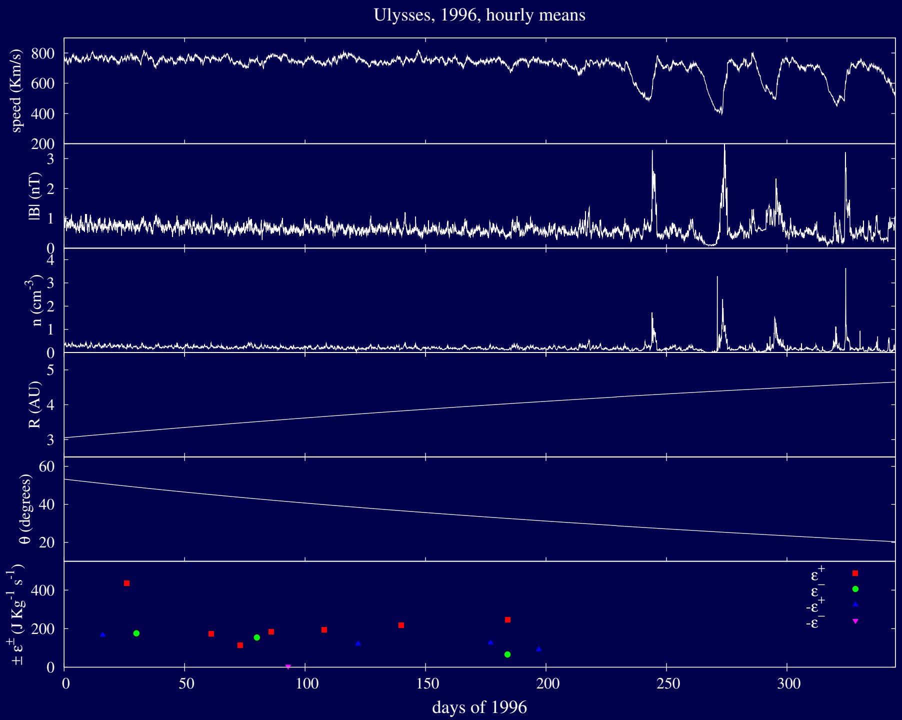
• Use 11 - days ($\approx$ 2000 data points) time-average moving window to build up statistical averages on stationary data sets, and avoid radial and latitudinal variations

Yaglom law is observed



First direct evidence of an MHD turbulent energy cascade in the solar wind

Measurements of the cascade rate



First measurements of the energy transfer rates $\varepsilon^{\pm} \approx 200\,\mathrm{J\,s^{-1}\,kg^{-1}}$

Effect of compressibility

- Incompressibility is not always satisfied in the solar wind

$$v/c_{\mathrm{s}} \approx v/c_{\mathrm{A}} \approx 5 - 10 \qquad \Rightarrow \qquad \rho \neq \mathrm{const}$$

Effect of compressibility

- Incompressibility is not always satisfied in the solar wind

$$v/c_{\rm s} \approx v/c_{\rm A} \approx 5-10 \qquad \Rightarrow \qquad \rho \neq \text{const}$$

$\Rightarrow$ the dissipation per unit volume $\varepsilon_V \equiv \rho\,\varepsilon = \rho\,u^3/\ell$
should be statistically constant
rather than the dissipation per unit mass ε

Effect of compressibility

- Incompressibility is not always satisfied in the solar wind

$$v/c_\mathrm{s} \approx v/c_\mathrm{A} \approx 5 - 10 \qquad \Rightarrow \qquad \rho \neq \mathrm{const}$$

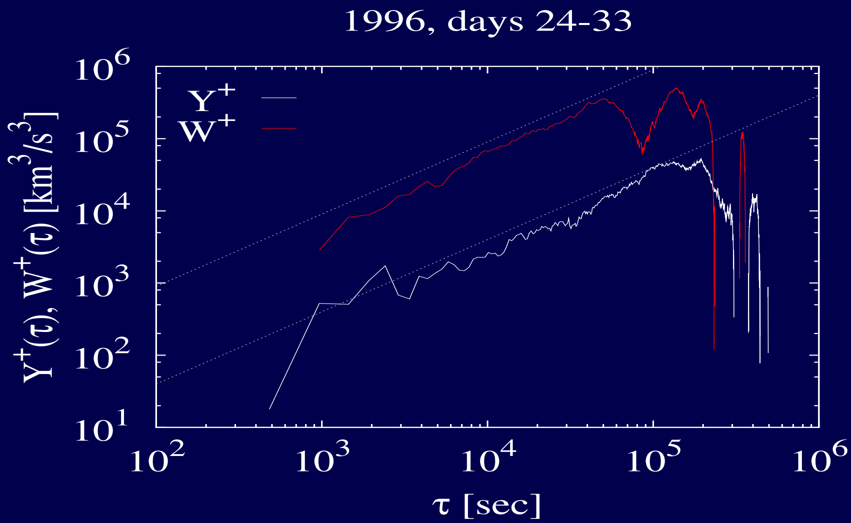
$\Rightarrow$ the dissipation per unit volume $\varepsilon_V \equiv \rho\,\varepsilon = \rho\,u^3/\ell$
should be statistically constant
rather than the dissipation per unit mass ε

$\Rightarrow$ Check for a Yaglom-type law using density-weighted Elsässer fields

$$\boldsymbol{w}^\pm \equiv \rho^{1/3}\,\boldsymbol{z}^\pm$$

and weighted flux $W^\pm(\ell) \equiv \dfrac{\left\langle |\Delta \boldsymbol{w}^\pm|^2 \Delta w_\parallel^\mp \right\rangle}{\langle\rho\rangle} \propto -\dfrac{4}{3}\,\varepsilon^\pm\,\ell$

Compressible scaling



Compressible scaling is observed

but the compressible pseudo-dissipations $\varepsilon_C^{\pm} \approx 3.5\,\mathrm{kJ\,s^{-1}\,kg^{-1}}$

are much larger than the incompressible dissipations $\varepsilon_I^{\pm} \approx 200\,\mathrm{J\,s^{-1}\,kg^{-1}}$

$\Rightarrow$ Correlations between density gradients and velocity/magnetic field

Solar wind heating

Solar wind radial temperature profile $T(R)$
decreases as a power law $T(R) \sim R^{-\xi}$ $\quad \xi \approx 0.7 - 1.0$
but slower than adiabatic spherical cooling $T(R) \sim R^{-4/3}$

Solar wind heating

Solar wind radial temperature profile $T(R)$

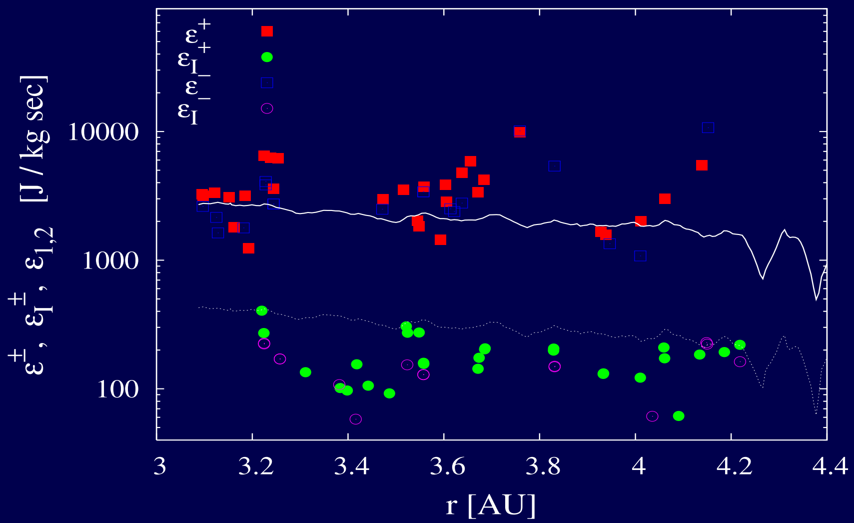
decreases as a power law $T(R) \sim R^{-\xi}$ $\qquad \xi \approx 0.7 – 1.0$

but slower than adiabatic spherical cooling $T(R) \sim R^{-4/3}$

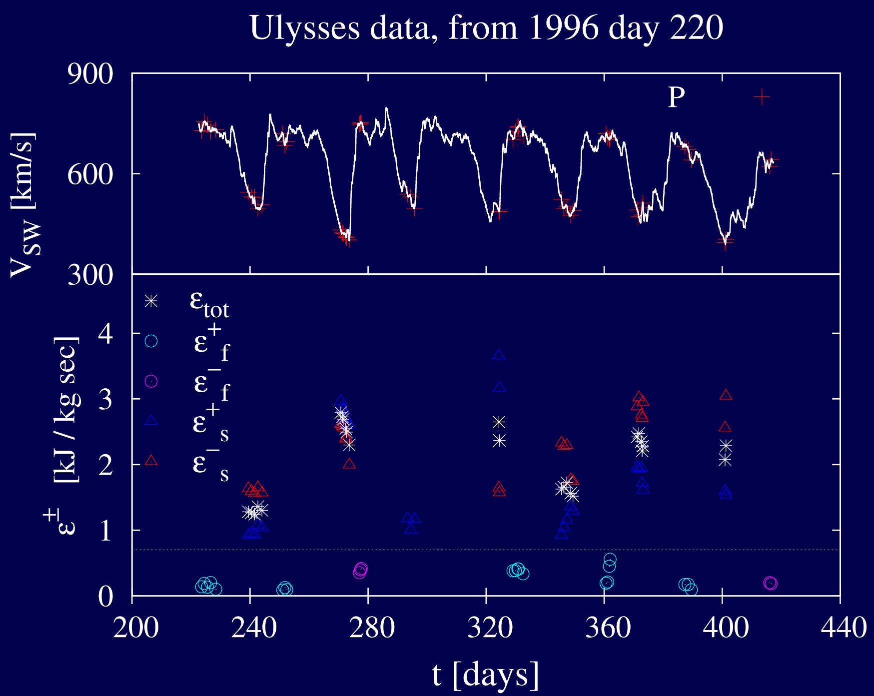
$\Rightarrow$ Necessity of in-situ heating . . . Turbulence ?

Solar wind heating

Solar wind radial temperature profile $T(R)$
decreases as a power law $T(R) \sim R^{-\xi}$ $\quad \xi \approx 0.7 - 1.0$
but slower than adiabatic spherical cooling $T(R) \sim R^{-4/3}$
$\Rightarrow$ Necessity of in-situ heating ... Turbulence ?



Measurements in the ecliptic



The dissipation rate ε depends on the type (fast/slow) of solar wind and is much higher for slow wind in the ecliptic

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
- Yaglom's law can be observed and is verified in turbulent MHD plasmas

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
- Yaglom's law can be observed and is verified in turbulent MHD plasmas
- The cascade transfer and dissipation rates $\varepsilon^{\pm}$ can be estimated

even without access to the dissipation scales of the flow

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
- Yaglom's law can be observed and is verified in turbulent MHD plasmas
- The cascade transfer and dissipation rates $\varepsilon^{\pm}$ can be estimated even without access to the dissipation scales of the flow
- Effects of compressibility are important

Conclusions

• The solar wind is a real turbulent magneto-hydrodynamic plasma

• Yaglom's law can be observed and is verified in turbulent MHD plasmas

• The cascade transfer and dissipation rates $\varepsilon^{\pm}$ can be estimated even without access to the dissipation scales of the flow

• Effects of compressibility are important

• Turbulence can account for (some of) the heating during the solar wind expansion

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
- Yaglom's law can be observed and is verified in turbulent MHD plasmas
- The cascade transfer and dissipation rates $\varepsilon^{\pm}$ can be estimated even without access to the dissipation scales of the flow
- Effects of compressibility are important
- Turbulence can account for (some of) the heating during the solar wind expansion

Thanks to everybody,
especially to the organizers . . .

Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
- Yaglom's law can be observed and is verified in turbulent MHD plasmas
- The cascade transfer and dissipation rates $\varepsilon^{\pm}$ can be estimated even without access to the dissipation scales of the flow
- Effects of compressibility are important
- Turbulence can account for (some of) the heating during the solar wind expansion

Thanks to everybody,
especially to the organizers . . .

and to Fabien !!!

Alain Arneodo † 2019

Fabien @ Cargèse 2007