Ocean rogue waves and the NLS equation

Christian Kharif

Institut de Recherche sur les Phénomènes Hors Equilibre (IRPHE)

Ecole Centrale Marseille

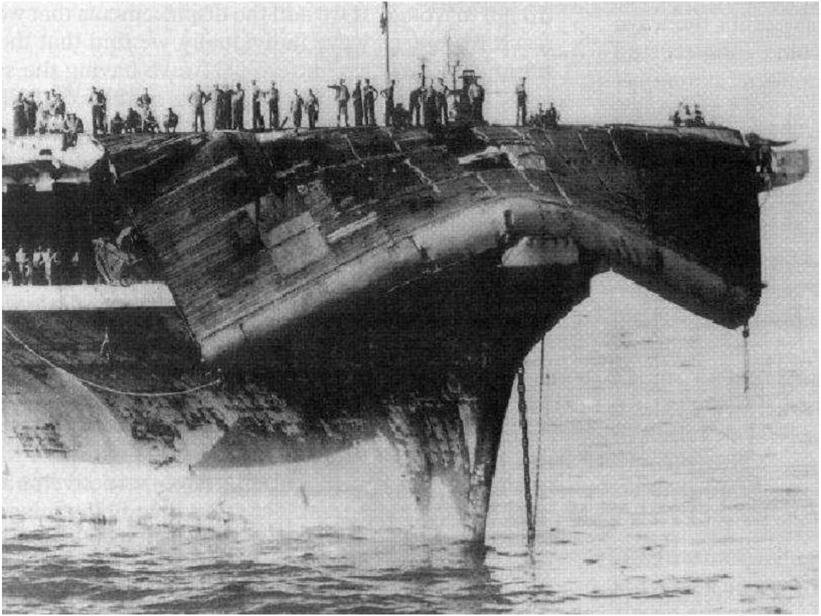
Observation of huge waves

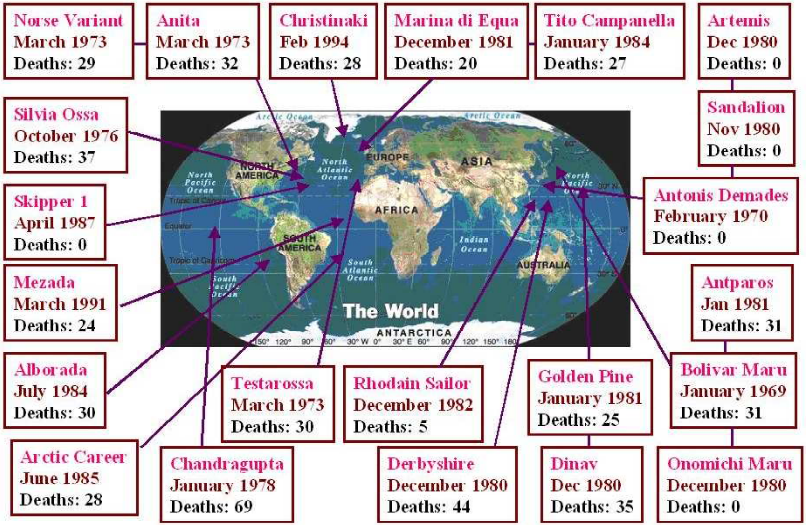
Photo of a huge wave event

Photo of a huge wave event

Photo of a huge wave event


WILSTAR





Super tanker collisions with freak waves (1968-1994).

Rogue waves in 2005 (Didenkulova *et al*, 2006)

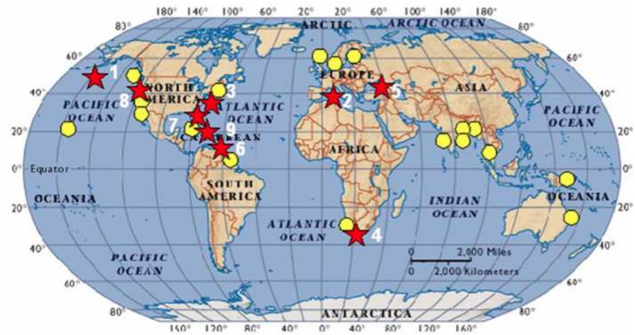
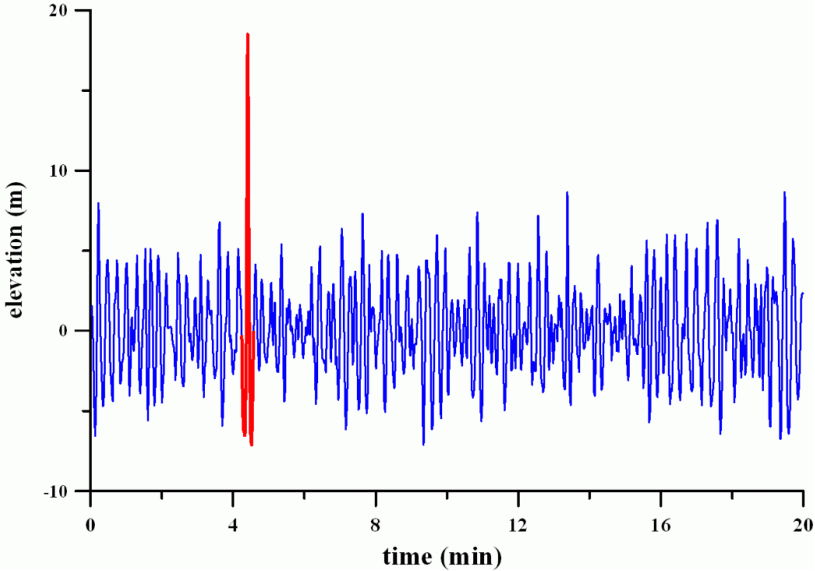


Fig. 1. Events selected as true freak waves are marked by red stars (1 – „Explorer", 2 – "Grand Voyager", 3 – "Norwegian Dawn", 4 – Kalk Bay, 5 – Blue Bay, 6 – Maracas Beach, 7 – Blake de Pastino, 8 – Port Orford, 9 – Petit Havre); yellow circles mark all other reported cases when abnormally large waves were observed.



- Time record of the New Year Wave in the North Sea ($H_{freak} = 25.3m$, $\eta_{crest} = 18.5m$, $H_s = 11.9m$)

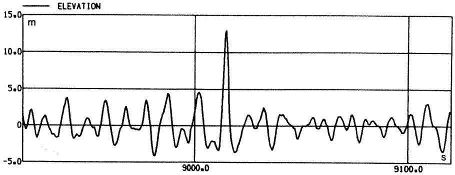


Figure 2. Freak wave A recorded on the 24th November, 1981 at the Gorm Field in the Danish Sector of the North Sea.

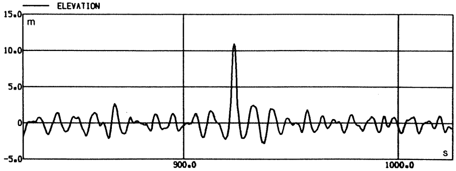


Figure 3. Freak wave B recorded on the 17th November, 1984 at the Gorm Field in the Danish Sector of the North Sea.

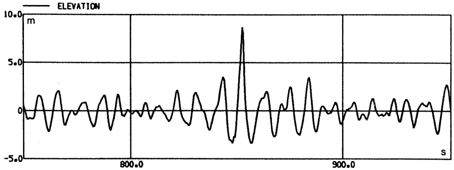


Figure 4. Freak wave C recorded on the 27th November, 1984 at the Gorm Field in the Danish Sector of the North Sea.

Ferry collisions with rogue waves near the French coast

- The Pont Aven ($L = 597\,ft$) hit a rogue wave ($\approx 15\,m$) during the night 21-22 May 2006.
- The Louis Majesty ($L = 207\,m$) hit a rogue wave ($\approx 17\,m$) on 3 March 2010 and two passengers were killed.
- The Jean Nicoli ($L = 200\,m$) hit a rogue wave (($\approx 20\,m$) on 6 March 2017.

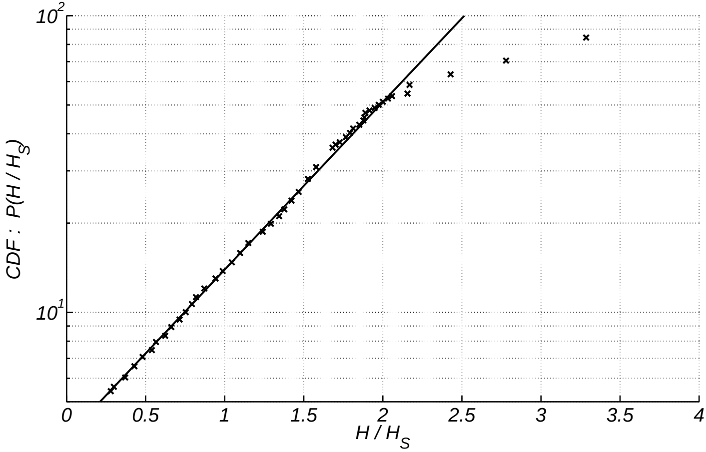
Physical mechanisms of rogue wave generation

- Wave-current interactions
- Geometrical or spatial focusing
- spatio-temporal focusing or dispersive focusing
- modulational instability (resonant four-wave interaction)
- crossing seas
- soliton collision
- etc.

- Mathematically, a rogue wave of height H_f satisfies

$$H_f > 2H_s$$

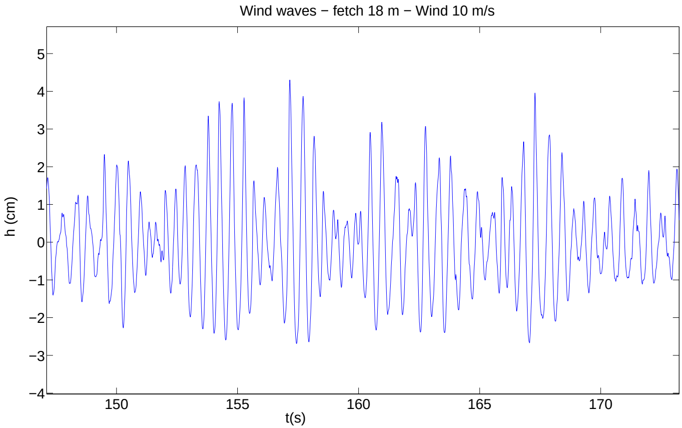
- H_f is more than twice the significant height H_s or 8 times the rms of the surface elevation
- Waves with larger heights than expected based on the Rayleigh distribution (abnormal waves)
- The above definition was proposed by Soren Peter Kjelsen in 1989 during the Workshop on Water Wave Kinematics, Molde (Norway)
- Water Wave Kinematics, Nato ASI series, eds. A.Torum & O.T. Gudmestad, 1990



Cumulative distribution function (%) as a function of normalized wave height H/H_s (from Sand *et al*, 1989) corresponding to North Sea data during stormy weather

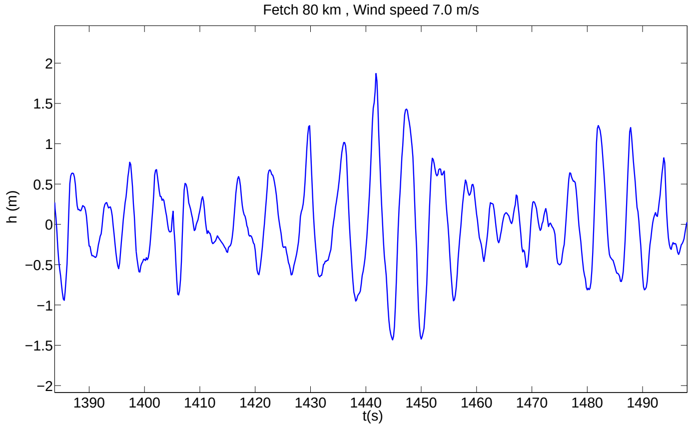
Modulated water waves without and with the presence of a shear current

Wind waves in the Large Air-Sea Interaction Facility (LASIF)



Fetch: $18\,m$ and wind speed: $10\,m \cdot s^{-1}$ from (H. Branger)

Wind waves in open field



Fetch: 80 km and wind speed: 7 $m \cdot s^{-1}$ (from H. Branger)

- This strong group structure of the wave field or modulational aspect of the surface elevation is due to resonant 4-wave interactions

- An elegant analytical method to study nonlinear modulational processes is the nonlinear Schrödinger equation

The nonlinear Schrödinger equation
(NLS equation)

- governs the spatio-temporal evolution of the complex envelope of the free surface of weakly nonlinear and dispersive water waves
- is a universal equation that can be derived from the nonlinear water wave equations using the method of multiple scales
- was first derived within the framework of water waves by Benney & Newell (1967)

- Let us consider a weakly nonlinear modulated wave train propagating on finite depth at the free surface of a shear current of constant vorticity

$$\eta(x,t) = \frac{1}{2}(\epsilon a(\xi,\tau)\exp[i(kx-\omega t)] + c.c) + \mathcal{O}(\epsilon^2)$$

where $\xi = \epsilon(x - c_g t)$ and $\tau = \epsilon^2 t$.

- Starting from the nonlinear water wave equations and using the method of multiple scales, the evolution of the complex envelope is governed by the NLS equation

$$i(a_\tau + c_g a_\xi) + L a_{\xi\xi} + N \mid a \mid^2 a = 0$$

$$L = \frac{\omega}{k^2\sigma(2+X)}\{\mu(1-\sigma^2)[1-\mu\sigma+(1-r)X]-\sigma r^2\}$$

$$N = -\frac{\omega k^2(U+VW)}{2(1+X)(2+X)\sigma^4}$$

$$U = 9-12\sigma^2+13\sigma^4-2\sigma^6+(27-18\sigma^2+15\sigma^4)X$$

$$+(33-3\sigma^2+4\sigma^4)X^2+(21+5\sigma^2)X^3+(7+2\sigma^2)X^4+X^5$$

$$V = (1+X)^2(1+r+\mu\overline{\Omega})+1+X-r\sigma^2-\mu\sigma X$$

$$W = 2\sigma^3\frac{(1+X)(2+X)+r(1-\sigma^2)}{\sigma r(r+\mu\overline{\Omega})-\mu(1+X)}$$

where $\mu = kh$, $\sigma = \tanh(\mu)$, $r = c_g/c_p$, $\overline{\Omega} = \Omega/\omega(\Omega)$ and $X = \sigma\overline{\Omega}$

In deep water without shear current the coefficients reads

$$L = -\frac{\omega}{8k^2} \quad \text{and} \quad N = -\frac{1}{2}\omega k^2$$

- The vor-NLS equation admits the following Stokes' wave solution

$$a = a_0 \exp(iNa_0^2\tau)$$

- Perturbation

$$a = a_0(1 + \delta a)\exp[i(\delta\omega + Na_0^2\tau)]$$

with

$$\delta a = (\delta a)_0 \exp[i(l\xi - \lambda\tau)]$$

$$\delta\omega = (\delta\omega)_0 \exp[i(l\xi - \lambda\tau)]$$

- Condition of linear instability with respect to sideband perturbations

$$LN > 0$$

$$\text{Let } L = L_1\omega/k^2 \text{ and } N = N_1\omega k^2$$

- Modulational instability occurs for perturbations whose wavenumber l satifies

$$-\sqrt{2\frac{N_1}{L_1}}ka_0 < \frac{l}{k} < \sqrt{2\frac{N_1}{L_1}}ka_0$$

- The growth rate of instability is

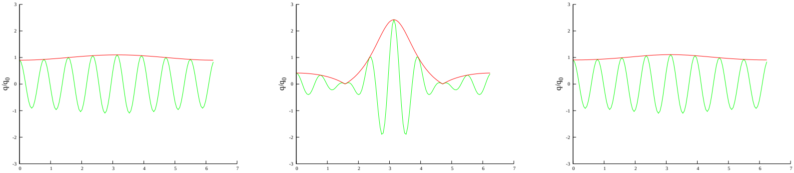
$$\gamma = \frac{l\omega}{k^2}\sqrt{2N_1L_1k^4a_0^2 - l^2L_1^2}$$

- Maximal growth rate

$$\gamma_{\max} = -N_1\omega(a_0k)^2 \quad \text{for} \quad l_{\max} = \sqrt{N_1/L_1}a_0k^2$$

- For $\Omega < -2\sqrt{\frac{gk}{3\sigma}} \Rightarrow$ no BF instability

Nonlinear evolution of BF instability

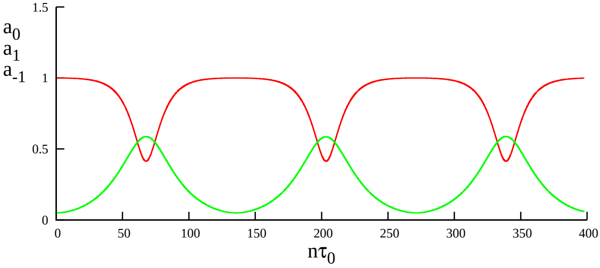


Envelope of surface elevation of a Stokes wave train of initial steepness $|A_0|k = 0.125$ amplified by its most unstable perturbation (FPU recurrence)

$$i\frac{\partial q}{\partial T} + \frac{\partial^2 q}{\partial X^2} + 2 \mid q \mid^2 q = 0$$

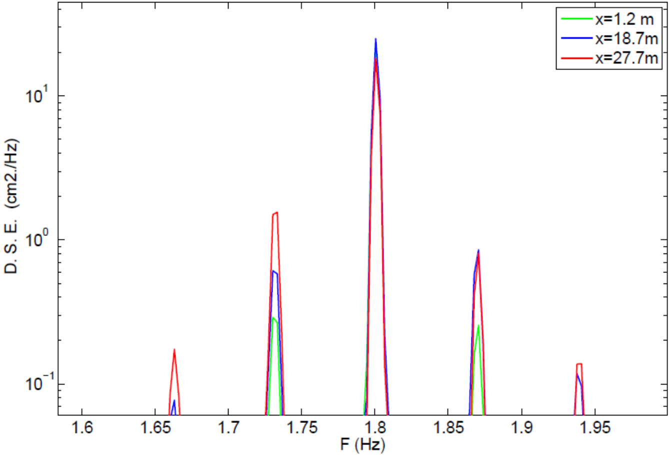
with

$$T = \frac{1}{2}\omega\tau, \qquad X = 2k\xi, \qquad q = \frac{1}{\sqrt{2}}kA^*$$



Temporal evolution of the carrier mode and satellites

Experimental spectrum evolution of modulated waves



Carrier wave steepness 0.07 (without wind, from H. Branger)

Application to Rogue Waves when modulational instability prevails

- The key parameter measuring the importance of the nonlinear four-wave interaction is the Benjamin-Feir Index (BFI) which is the ratio of the wave steepness to the normalized spectral bandwidth.

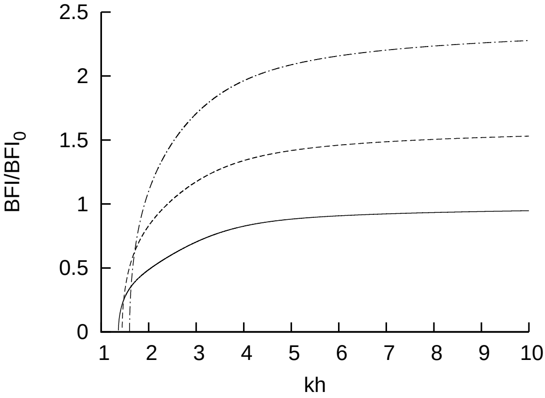
- Within the framework of the NLS equation the BFI writes

$$BFI = \frac{a_0 k}{\Delta K / k} \sqrt{\mid N_1 / L_1 \mid}$$

where ΔK is a typical spectral bandwidth (Onorato *et al*, 2006 and Kharif *et al*, 2009)

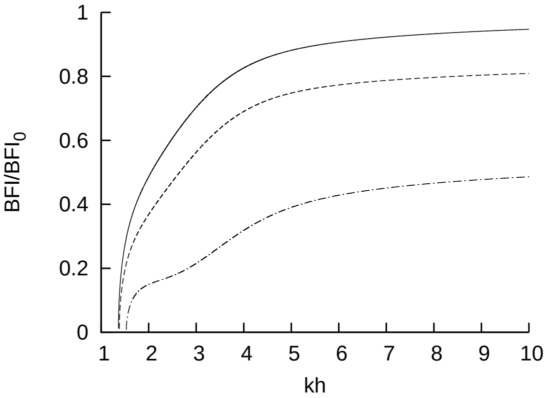
- The BFI is a convenient indicator for prediction of rogue wave occurrence. It is related to the pdf of wave heights. The rogue wave probability occurrence increases with the BFI.

Thomas, Kharif & Manna (POF, 2012)

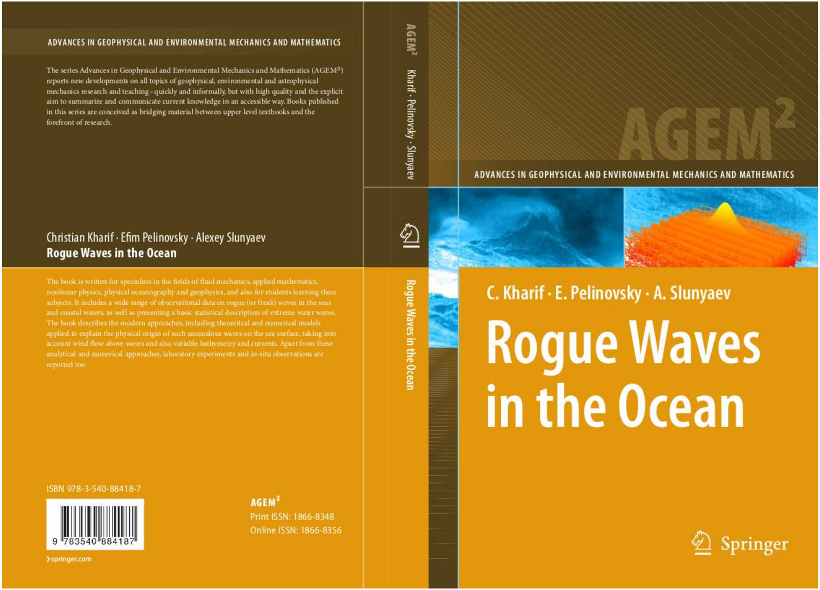


Normalized BFI as a function of *kh* for several values of $\overline{\Omega}$:
$\overline{\Omega} = 0$ (solid line), $\overline{\Omega} = 1$ (dashed line), $\overline{\Omega} = 2$ (dot-dashed line)

Thomas, Kharif & Manna (POF, 2012)



Normalized BFI as a function of kh for several values of $\overline{\Omega}$: $\overline{\Omega} = 0$ (solid line), $\overline{\Omega} = -0.3$ (dashed line), $\overline{\Omega} = -0.6$ (dot-dashed line)


ADVANCES IN GEOPHYSICAL AND ENVIRONMENTAL MECHANICS AND MATHEMATICS
The series Advances in Geophysical and Environmental Mechanics and Mathematics (AGEM²) reports new developments on all topics of geophysical, environmental and astrophysical mechanics research and teaching - quickly and informally, but with high quality and the explicit aim to summarize and communicate current knowledge in an accessible way. Books published in this series are conceived as bridging material between upper level textbooks and the forefront of research.
Christian Kharif · Efim Pelinovsky · Alexey Slunyaev
Rogue Waves in the Ocean
The book is written for specialists in the fields of fluid mechanics, applied mathematics, nonlinear physics, physical oceanography and geophysics, and also for students learning these subjects. It includes a wide range of observational data on rogue (or freak) waves in the seas and coastal waters, as well as presenting a basic statistical description of extreme water waves. The book describes the modern approaches, including theoretical and numerical models applied to explain the physical origin of such anomalous waves on the sea surface, taking into account wind flow above waves and also variable bathymetry and currents. Apart from these analytical and numerical approaches, laboratory experiments and in-situ observations are reported too.
ISBN 978-3-540-88418-7
springer.com
AGEM²
Print ISSN: 1866-8348
Online ISSN: 1866-8356
AGEM² Kharif · Pelinovsky · Slunyaev
Rogue Waves in the Ocean
AGEM²
ADVANCES IN GEOPHYSICAL AND ENVIRONMENTAL MECHANICS AND MATHEMATICS
C. Kharif · E. Pelinovsky · A. Slunyaev
Rogue Waves in the Ocean
Springer