Secondary instability in variable density Jets

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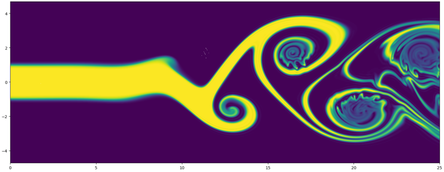


Environmental and industrial applications

Mathematical formulation

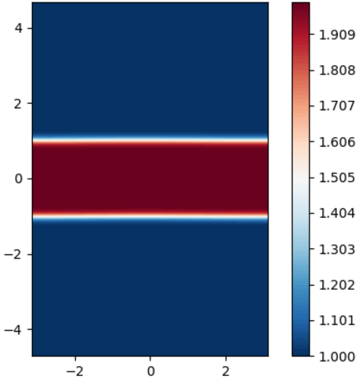
- Variable density Navier–Stokes equations:

$$
\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} &= -(\mathbf{u}\cdot\nabla)\,\mathbf{u} - \frac{\nabla p}{\rho} + \zeta(\rho,\mathbf{u}) + \mathbf{f},\\
\frac{\partial \rho}{\partial t} &= -\mathbf{u}\cdot\nabla\rho + \frac{1}{R_e S_c}\Delta\rho,\\
\nabla\cdot\mathbf{u} &= 0,\\
\zeta(\rho,\mathbf{u}) &= \frac{1}{\rho R_e}\Delta\mathbf{u} + \frac{1}{\rho R_e S_c}\left[(\mathbf{u}\cdot\nabla)\,\nabla\rho + (\nabla\rho\cdot\nabla)\mathbf{u}\right],
\end{aligned}
$$

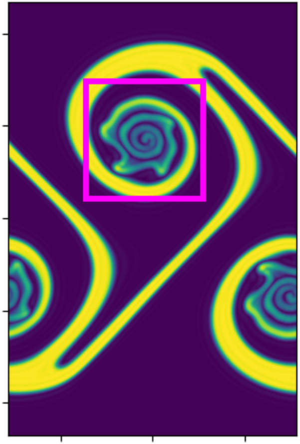
- Direct numerical simulation :
  - Spectral decomposition
  - Semi-implicit second order time accurate
  - Preconditionned iterative solver (pressure, update velocity)

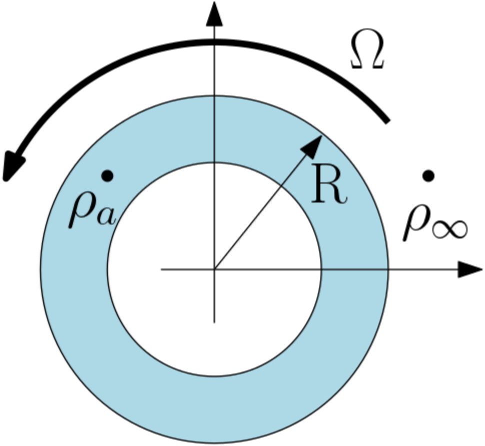
Numerical simulation (DNS)

- $Re_{jet} = \rho_\infty U_{jet} L_{jet}/\mu = 1000, \; Sc = \mu/\kappa = 10, \; s = \rho_{jet}/\rho_\infty = 2$



Secondary instability: idealised flow





Dynamic equation : Inviscid ($Re \to \infty$)

- Modal decomposition $f(r,\theta,t) = f(r)e^{i(m\theta-\omega t)}$
- Dynamic equation : $\phi = ru$

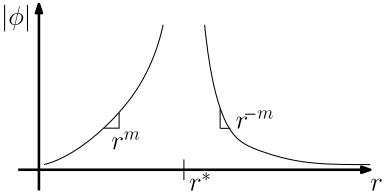
$$\frac{d^2\phi}{dr^2}+\frac{1}{r}\left(1-\frac{G^2}{\Omega^2}\right)\frac{d\phi}{dr}-\frac{1}{r^2}\left[-\frac{2mG^2}{\Omega\Sigma}+m^2\left(1+\frac{G^2}{\Sigma^2}\right)\right]\phi = 0,$$

with

$$G^2 = -\frac{\Omega^2 r}{\rho_0}\frac{d\rho_0}{dr}, \qquad \Sigma = m\Omega - \omega.$$

Far field solution

- Far from density gradients ($G^2 \to 0$)



- ϕ admit at least one extremum near r^*
- Assumption :

$$\frac{dG^2}{dr}(r^*) = 0 \quad \text{and} \quad \frac{d\phi}{dr}(r^*) = 0$$

Dispersion Relation

- Solution : $\phi(r) = H_n(\alpha\tilde{r} + \beta)$ (Weber-Hermite polynom) if

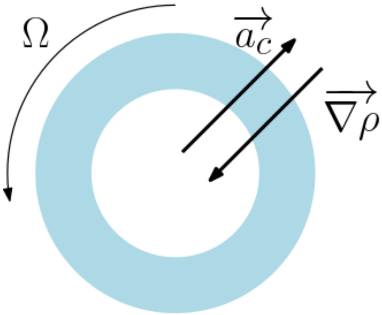
$$2\chi^2(r^*) + \frac{\chi(r^*)r^{*2}}{2}\frac{d^2\chi}{dr^2}(r^*) + (2n+1)\left(3\chi(r^*) + \frac{r^{*2}}{2}\frac{d^2\chi}{dr^2}(r^*)\right)^{3/2} = 0$$

with

$$\chi(r) = -2m\frac{G^2}{\Omega\Sigma} + m^2\left(1 + \frac{G^2}{\Sigma^2}\right)$$

The instability

- Existance of an unstable mode if $G^2(r^*) > 0$!



$\Rightarrow$ Cylindrical Rayleigh–Taylor instability

Asymptotic viscous correction

- Asymptotic limit : $Re \gg 1,\ m \gg 1,\ m^2/Re \ll 1$

$$\frac{d^2\phi}{dr^2} + \zeta(r,\rho_0,m,Re)\frac{d\phi}{dr} + \lambda(r,\rho_0,G^2,m,Re)\phi = 0$$

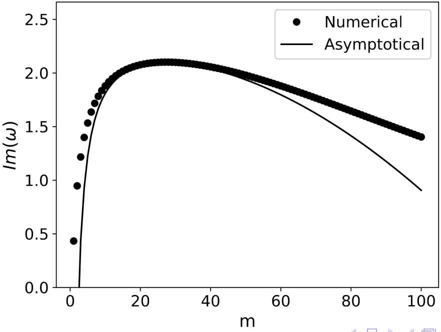
- Asymptotic viscous correction

$$\omega_{vis} = \omega_{inv} - \frac{m^2}{2Re\rho_0(r^*)r^{*2}}$$

Numerical validation: dispersion relation

- Eigenvalue solver

$$\mathbb{A}\begin{pmatrix}u\\v\\\rho\end{pmatrix}=\omega\begin{pmatrix}u\\v\\\rho\end{pmatrix},\quad \text{with}\quad p=\left(\nabla\cdot\frac{1}{\rho_0}\nabla\right)^{-1}f(u,v,\rho)$$



Most unstable mode

- Maximum growth rate:

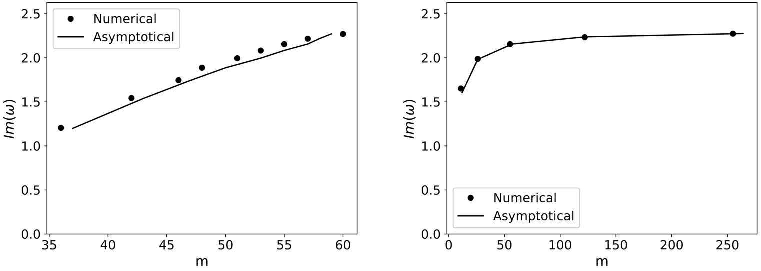
$$\omega_{n,max} = m\Omega + \frac{G^2(r^*)}{\Omega m} + i\sqrt{G^2} - \frac{3i}{2}\frac{\left(\left(n+\frac{1}{2}\right)\sqrt{-\frac{r^{*2}}{2}\frac{d^2G^2}{dr^2}(r^*)}\right)^{\frac{2}{3}}}{\left(r^{*2}\rho_0(r^*)Re\right)^{\frac{1}{3}}}$$

- Corresponding azimutal wave number:

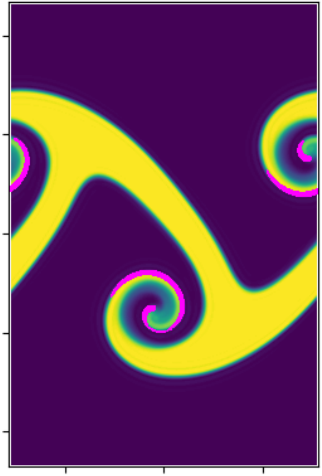
$$m_{max} = \left(\left(n+\frac{1}{2}\right)Re\rho_0(r^*)r^{*2}\right)^{\frac{1}{3}}\left(-\frac{r^{*2}}{2}\frac{d^2G^2(r^*)}{dr^2}\right)^{\frac{1}{6}}$$

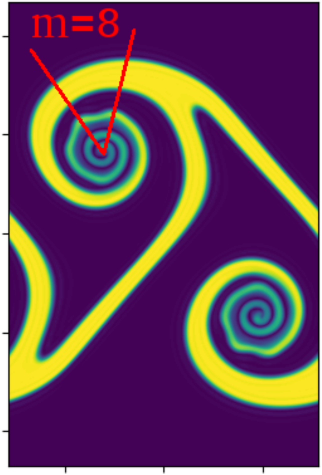
Numerical validation: most unstable mode

- $Re = 10^4, \quad s = 2 \ldots 10$
- $s = 8, \quad Re = 10^2 \ldots 10^6$



Comparison with DNS





Perspectives

- Weakly non-axisymetric (WKB)
- Three dimensionnal instability (theoretical, DNS)
- Turbulence transition (DNS, by pass ?)