Universality of some turbulence laws versus Finite Reynolds Number (FRN) effects

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Outline

- What is the relevant high Reynolds number limit? Two examples:
- (I) The 4/5 Kolmogorov law. Unstructured modelling for HIT
- (II) Small scale anisotropy. Partly structured modelling for USHT (Unstable Stratified Homogeneous Turbulence)

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- What is the relevant high Reynolds number limit? Two examples:
- (I) The 4/5 Kolmogorov law. Unstructured modelling for HIT
- (II) Small scale anisotropy. Partly structured modelling for USHT (Unstable Stratified Homogeneous Turbulence)
- Some related questions on internal intermittency and anomalous exponents
- Conclusions and future prospects about the use of multimodal, possibly anisotropic, holistic triadic spectral closure, EDQNM and beyond

What are really very high — or very low — nondimensional numbers

Some outstanding nondimensional numbers, from my limited experience

- The magnetic Prandtl number in some liquid metals: $10^{-7} - - - 10^{-8}$ (ratio of molecular diffusivities)
- The Poincaré number in rotating flows with precession: e.g. 10^{-7} for the Earth since Ptolemeus (ratio of angular velocities)

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Scale-dependent numbers, from Rayleigh to Reynolds

- The Rayleigh number in Rayleigh / Bénard convection: 10^{13} and beyond?
- The Reynolds number, here defined as a micro, or Taylor-based one Re_λ

Not to mention several numbers in internal and external geophysics, and astrophysics.

Purely HIT. FRN for the 4/5-Kolmogorov law: Unstructured Modelling

From *isotropized* K-H equation (revisited, E. Lindborg, F. Moisy),

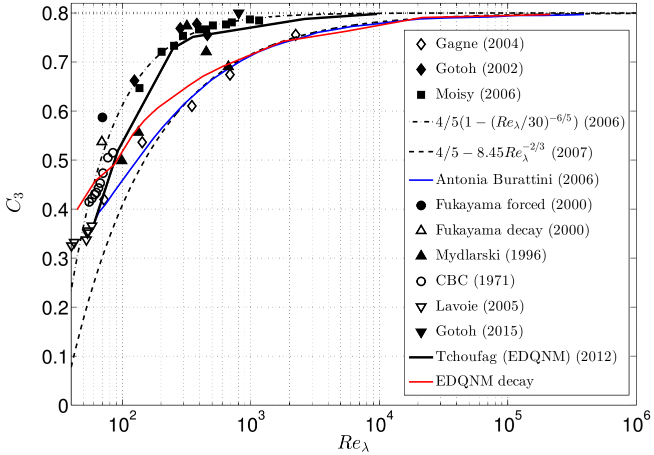
$$S_3(r,t) = -\frac{4}{5}\varepsilon(t)r - \frac{3}{r^4}\int_0^r r^4 \frac{\partial S_2(r,t)}{\partial t} dr + 6\nu \frac{\partial S_2(r,t)}{\partial r},$$

to extract a FRN-dependent parameter C_3 (Antonia & Burattini 2006) to be plotted in several physical / numerical exps. $R_{LL,L}(r,t) \to S_3(r,t)$

$$C_3 = -\max_r S_3^*(r), \quad S_3^*(r) = \frac{S_3(r)}{-\varepsilon r}$$

- $C_3 = 4/5$ in the asymptotic limit of very high Reynolds number, discarding unsteady and *viscous* effects in the K-H equation.
- An opportunity to evaluate what means *very high Reynolds number* and to re-open an old debate on *anomalous exponents*, with ESS and so on $\cdots$

From Antonia & Burrattini (2006), Tchoufag *et al.* (2012), and Sagaut & CC monograph (2018)



FRN modelling, focus on the decaying case only $\cdots$ and naive remarks

- A huge Taylor-based Reynolds number R_λ for asymptotic validity of 4/5-K law: 5.10^4
- FRN effects well fitted at the highest R_λ ($R_\lambda \geq 10^3$.) EDQNM (rerun from Antoine Briard after Tchoufag *et al.*, 2012) agrees vs. all the fits., the simplest being from Lindborg using K42 for $S_2(r,t) = C_2(\varepsilon(t)r)^{2/3}$ for unsteady (with $k-\varepsilon$) and viscous terms in K-H.
- FRN at lower R_λ: Really new results from EDQNM vs. all fits! CBC (1971) points were added using $T(k,t) \leftrightarrow S_3(r,t)$

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i) what is the *more exact*, $S_2 \sim 2(\varepsilon r)^{2/3}$ or $S_3 = -\frac{4}{5}\epsilon r$?? the Lindborg's fit, namely, is at odds with several papers on anomalous exponents, using Extended Self Similarity,
ii) What is the meaning of *error bars* used by the latter community for assessing anomalous exponents ξ_n for S_n, $n \neq 3$, with only $\xi_3 = 1$?

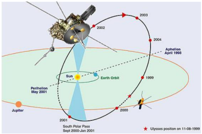
From Raffaele Marino: advocating for (4/5) – (4/3)-K laws in astrophysics

Extension of the 4/5 - law to MHD : observation in the Solar Wind

Politano & Pouquet law (MHD)

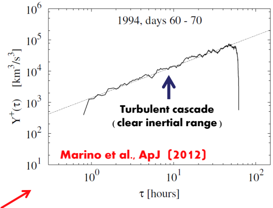
$$Y^{\pm} = \left\langle \Delta z_{\ell}^{\mp} \left(\Delta z_i^{\pm} \right)^2 \right\rangle = -\frac{4}{3} \varepsilon^{\pm} \ell$$

Geophys. Res. Lett., Vol. 25, p. 273 (1998)



Firsts estimates of ε in space plasmas !

Ulysses (esa/NASA)



P&P law for compressible MHD

$$W^{\pm} = \left\langle \Delta z_{\ell}^{\mp} \rho^{1/3} \left| \Delta \underline{z}^{\pm} \rho^{1/3} \right|^2 \right\rangle \cdot \langle \rho \rangle^{-1} = -\frac{4}{3} \varepsilon^{\pm} \ell$$

Marino, Sorriso-Valvo, Carbone, Noullez, Bruno et al. :

Phys. Rev. Lett., Vol. 99, p. 115001-1 (2007)
Astrophys. J. , Vol. 667, p.L71 (2008), Vol. 750:41 (2012)
Phys. Rev. Lett. , Vol. 103, p.061102 (2009)
Phys. Rev. Lett. , Vol. 104, p.189002, (2010)
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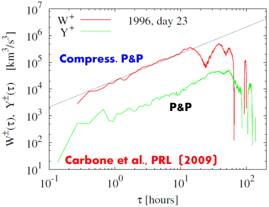
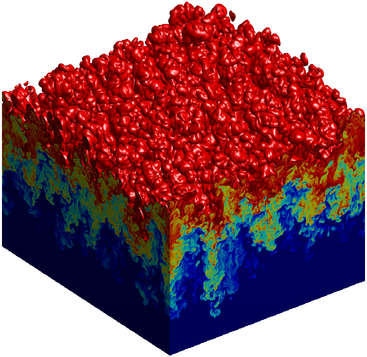
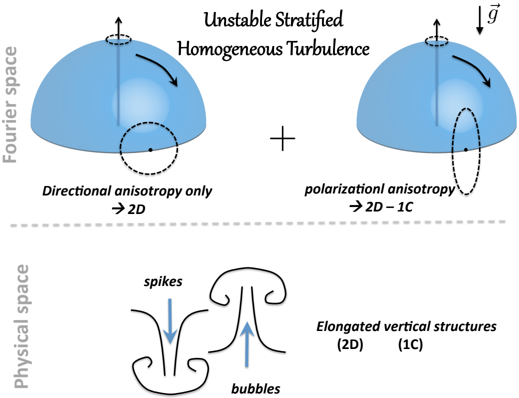


Second example, highy anisotropic USHT as homogenized RTT



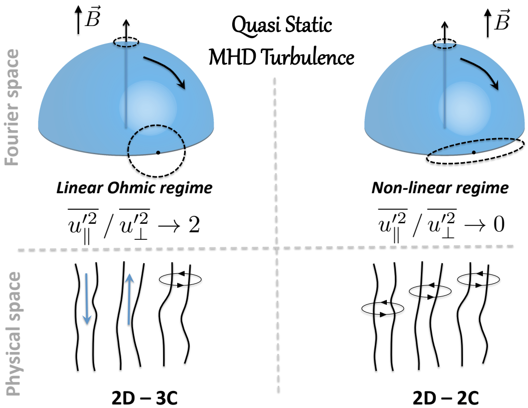
From Benoît-Joseph Gréa, CEA, RTT, high-resolution DNS by triclade.

Both directional and polarization anisotropy in USHT



(Sagaut & CC, 2018, Chap 10; Burlot *et al.*, JFM, PoF, 2015)

Similar directional, opposite polarization, QSMHD



(Sagaut & CC, 2018, Chap. 12, Favier *et al.*, JFM, 2011)

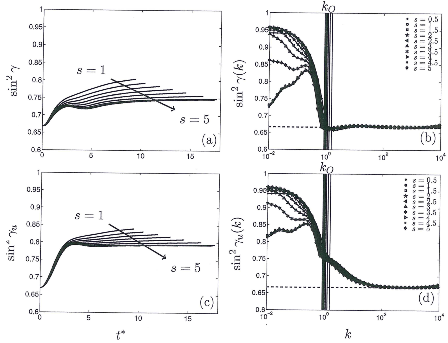
USHT, collaboration LMFA with CEA, anisotropy at really high Reynolds number

- Relevance of multimodal anisotropic EDQNM for USHT, first cross-validated with DNS at the highest Reynolds number available, then using only generalized EDQNM at really huge Reynolds number
- Quantifying both directional and polarization anisotropies for an unprecented range of Reynolds numbers.

$$\sin^2 \gamma_u = \frac{\iiint \sin^2 \theta_k \mathcal{E}(\boldsymbol{k}, t) d^3 \boldsymbol{k}}{\iiint \mathcal{E}^{(pot)}(\boldsymbol{k}, t) d^3 \boldsymbol{k}}$$

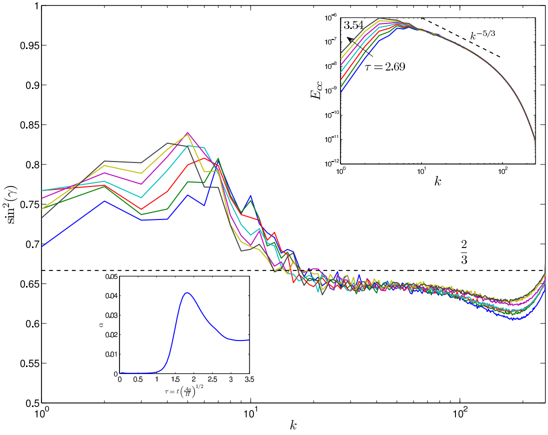
$$\sin^2 \gamma = \frac{\iiint \sin^2 \theta_k \mathcal{E}^{(pot)}(\boldsymbol{k}, t) d^3 \boldsymbol{k}}{\iiint \mathcal{E}^{(pot)}(\boldsymbol{k}, t) d^3 \boldsymbol{k}}$$

After DNS, only EDQNM for huge R_e, from Burlot *et al.*, PoF, 2015



axisym, production by N, similar threshold scales, $\sqrt{S^3/\varepsilon}$ (Corrsin), Ozmidov (N), Zeman (Ω). Isotropy for $k \geq k_{Ozmidov}$ is only achieved at huge Reynolds number with EDQNM.

Parameter of directional dependence even in inhomogeneous DNS calculations



From CC & Gréa, JoT (2013)

General closure strategy, with — and without, USHT — waves

Transferring the machinery of EDQNM from $\hat{\boldsymbol{u}}$ to *slow* amplitudes.

- The typical equation for three-point third-order correlations to be closed:

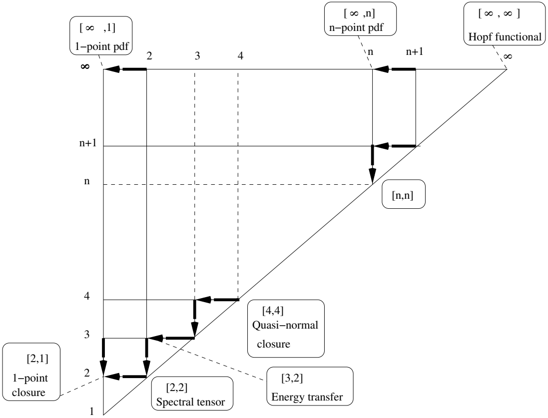
$$\left(\frac{\partial}{\partial t} + \nu(k^2 + p^2 + q^2) + \imath(s\sigma(\boldsymbol{k}) + s'\sigma(\boldsymbol{p}) + s''\sigma(\boldsymbol{q}))\right) S_{ss's''}(\boldsymbol{k}, \boldsymbol{p}, t) =$$

$$= T^{(QN)}_{ss's''} + C^{(IV)}_{ss's''}, \quad s, s', s'' = 0, \pm 1, \quad \boldsymbol{k} + \boldsymbol{p} + \boldsymbol{q} = \boldsymbol{0}$$

- Classical approach to wave turbulence,
  QN ($C^{IV} = 0, \langle vvvv \rangle = \sum \langle vv \rangle \langle vv \rangle$) (*e.g. Benney and Newell, 1969*)
  $\leftrightarrow$ Random Phase Approximation
  Markovianisation $\leftrightarrow$ two time-scales t and ϵt, final equations in terms of slow variables only.
- Including an additional Eddy Damping ingredient as in EDQNM for HIT, for the zero mode ($s = s' = s'' = 0$).

$$C^{(IV)} = -(\eta(k) + \eta(p) + \eta(q)) S_{000}.$$

Hierarchies for statistical closures, third-order correlations *at three points*!



Generalized EDQNM and beyond?

- A general strategy, not a new theory, equations not carved in the marble. To be matched with Wave-Turbulence theory.
- Possibility to take into account detailed anisotropy, including directional one connected to dimensionality , from 3D to 2D, 1D. Effects of mean gradients, body forces: not a perturbative approach, without formal expansion around isotropy as in (Kraichnan's legacy, DIA, LHDIA, TFM, LRA ... etc)

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- Fully numerical solution, with quantitative comparison with DNS at highest resolution (CC et al., JFM 1997, Burlot et al., JFM 2015) Integration over the orientation of triads: fully numerical (from CC & Jacquin 1989, Bellet *et al.* 2006) to semi-analytical (but with truncated anisotropy) with Mons *et al.* 2016, Ying *et al.* (2019).
- An unprecedented investigation of the finite Reynolds number effect, initial data, and parametric study in general.