Is small-scale turbulence really "anomalous" ?

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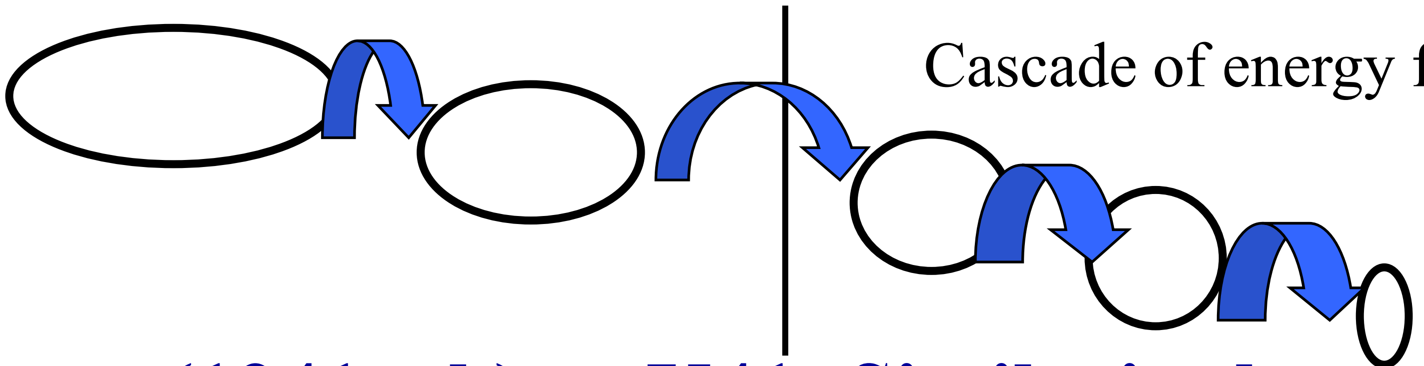
Many Happy Returns to Fabien



Kolmogorov (1941a, b) or K41: Similarity hypotheses (very HIGH Reynolds numbers)

1st: pdf of $\delta u = u(x+r) - u(x)$ is unambiguously defined by ν and $\bar{\epsilon}$ (also Batchelor 1947) .

$$\overline{(\delta u^*)^n} = f_{un}(r^*) \quad r^* = r/\eta \quad \eta = (\nu^3/\bar{\epsilon})^{1/4} \quad u_K = (\nu\bar{\epsilon})^{1/4}$$

2nd: In the IR $\eta \ll r \ll L$ (L is the integral length scale), pdf of $\delta u/(\bar{\epsilon} r)^{1/3}$ is universal. $\overline{(\delta u^*)^n} = C_{un} r^{*n/3}, \quad C_{un} = \text{"}universal\text{" constants}, \ C_{u3} = -4/5$

K62--Kolmogorov (1962), Obukhov (1962)

Kolmogorov's third hypothesis or RSH

To account for the "internal intermittency" or spatio-temporal randomness of ε (e.g. Batchelor & Townsend 1949) ? Or to account for the non-stationarity associated with large scales ?

(i) $\varepsilon_r = \frac{1}{r}\int_0^r \varepsilon(x+h)dh$

(ii) pdf of ε_r is "lognormal" in the IR $\sigma^2_{\ln\varepsilon_r} = A + \mu \ln\frac{L}{r}$

$$\overline{|\delta u|^n} \sim (\overline{\varepsilon} r)^{n/3} \left(\frac{L}{r}\right)^{\frac{\mu n(n-3)}{18}} \qquad \overline{|\delta u|^n} \sim r^{\zeta_{un}} \quad \text{where} \quad \zeta_{un} = \frac{n}{3} - \frac{\mu n(n-3)}{18}$$

Yaglom (1966) $\overline{\varepsilon(x)\varepsilon(x+r)} \sim \overline{\varepsilon}^{2}\left(\frac{L}{r}\right)^{\mu}$ in the IR

Meneveau Sreenivasan 1987

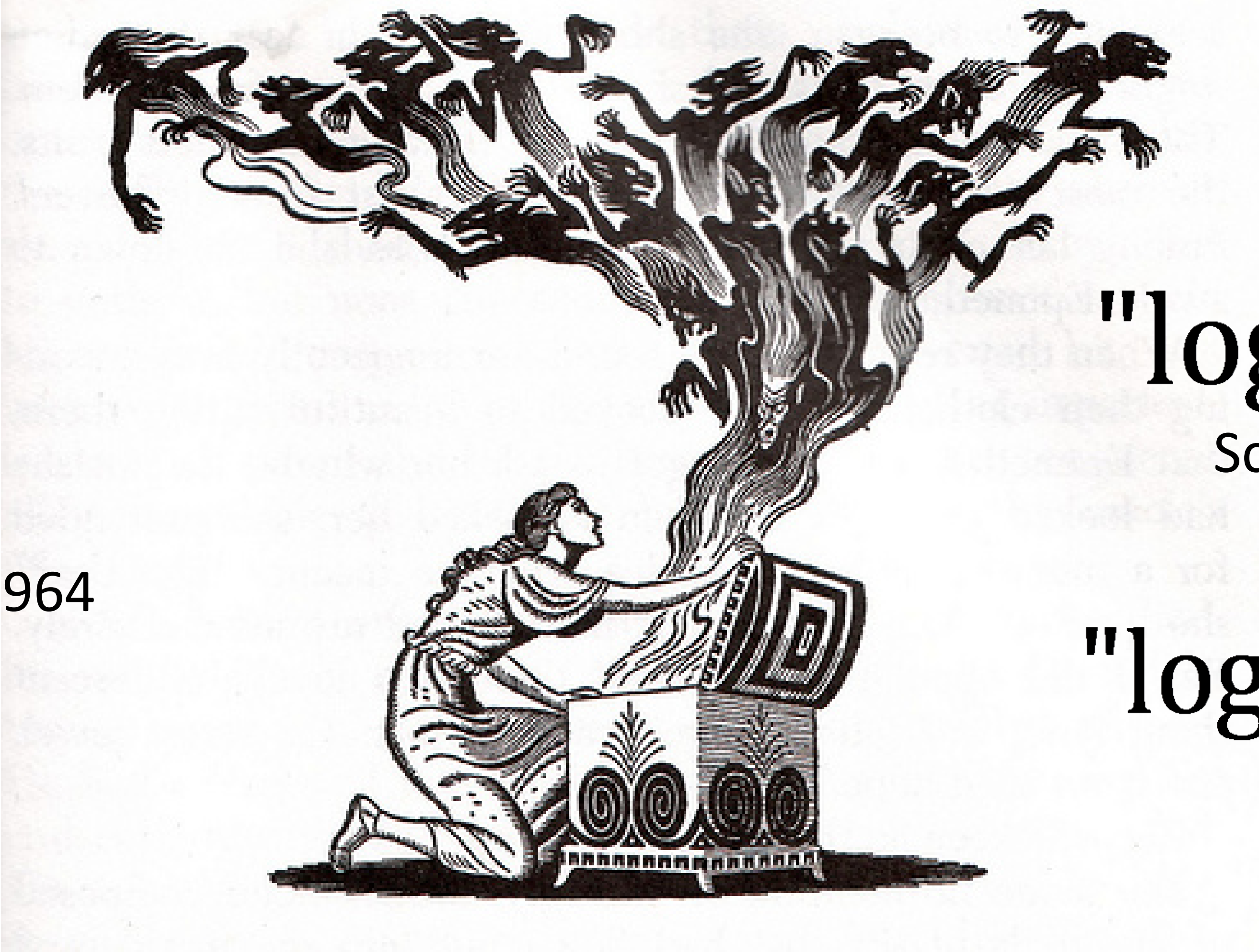
"p"

"β"

Frisch et al 1978

Novikov Stewart 1964

"log *normal*" Kolmogorov 1962 Obukhov 1962



"log – *Lévy*"

Schertzer Lovejoy 1992

"log – *Poisson*"

She Leveque 1994

K62 has been described as opening a **Pandora's box** of possibilities
e.g. Kraichnan (1974), Saffman (1977), Moffatt (1994), Davidson (2004)

Karman-Howarth (1938) equation (HIT)

$$-\frac{1}{3}\left(\frac{\partial}{\partial r}+\frac{4}{r}\right)\overline{(\delta u)^3}=\frac{4}{3}\bar{\varepsilon}-2\nu\left(\frac{\partial^2}{\partial r^2}+\frac{4}{r}\frac{\partial}{\partial r}\right)\overline{(\delta u)^2}+\frac{\partial}{\partial t}\overline{(\delta u)^2}$$

Integrate with respect to r to yield **"scale-by-scale (or s-b-s) energy budget"** **(Danaila Anselmet Zhou Antonia 1999)**

$$-\overline{(\delta u)^3}=\frac{4}{5}\bar{\varepsilon}r-6\nu\frac{\partial}{\partial r}\overline{(\delta u)^2}$$

Large scale term

$$+\frac{3}{r^4}\int_0^r s^4\frac{\partial}{\partial t}\overline{(\delta u)^2}ds\,(\equiv I_u)$$

If $\partial / \partial t$ is of order $\bar{\varepsilon} / \overline{u^2}$, $\partial\overline{(\delta u)^2} / \partial t$ is negligible provided

$$\frac{\overline{(\delta u^*)^2}}{R_\lambda} \ll 1 \quad \text{or} \quad \frac{\overline{(\delta u)^2}}{\overline{u^2}} \ll 1$$

These requirements are met if R_λ $(\equiv u'\lambda / \nu)$ is very large or $r \ll L$

Kolmogorov assumed that R_λ is very large and obtained

$$-\overline{(\delta u)^3} = \frac{4}{5}\bar{\varepsilon} r - 6\nu \frac{\partial}{\partial r}\overline{(\delta u)^2}$$

In the IR ν can be neglected

A "sine qua non" condition before testing the 2nd hypothesis

$$-\overline{(\delta u)^3} = \frac{4}{5}\bar{\varepsilon} r$$

Kolmogorov's "4/5" law

(Why not the "1" law ?)

Antonia Djenidi Danaila (2014)

$$\overline{(\delta u)^2} = u_0^2 f\left(\frac{r}{l_0}\right) \qquad \overline{(\delta u)^3} = u_0^3 g\left(\frac{r}{l_0}\right)$$

$$\frac{u_0^3}{\bar{\varepsilon} l_0} g\left(\frac{r}{l_0}\right) = \frac{4}{5}\frac{r}{l_0} - 6\left(\frac{u_0^3}{\bar{\varepsilon} l_0}\right)\left(\frac{\nu}{u_0 l_0}\right) f' \qquad \text{Eq. (1)}$$

Similarity is satisfied if $\frac{\bar{\varepsilon} l_0}{u_0^3} = C_1$ and $\frac{u_0 l_0}{\nu} = C_2$

C_1 and C_2 are equal to 1 if $l_0 \equiv \eta$ and $u_0 \equiv u_K$

Eq. (1) does NOT require R_λ to be large for the dissipative scales to satisfy similarity

Common feature between K41 and K62

$$\overline{(\delta u)^3} = -\,(4/5)\,\overline{\varepsilon}\,r$$

$$i.e.\quad \zeta_{u3} = 1$$

Some Implications of K62

$$\overline{(\delta u^*)^2} \sim r^{*\frac{2}{3}+\frac{\mu}{9}} R_\lambda^{-\frac{8\mu}{6}}$$

$$\overline{(\delta u^*)^2} \to 0 \qquad\qquad R_\lambda \to \infty$$

$$-\overline{(\delta u)^3} = \frac{4}{5}\overline{\epsilon} r$$

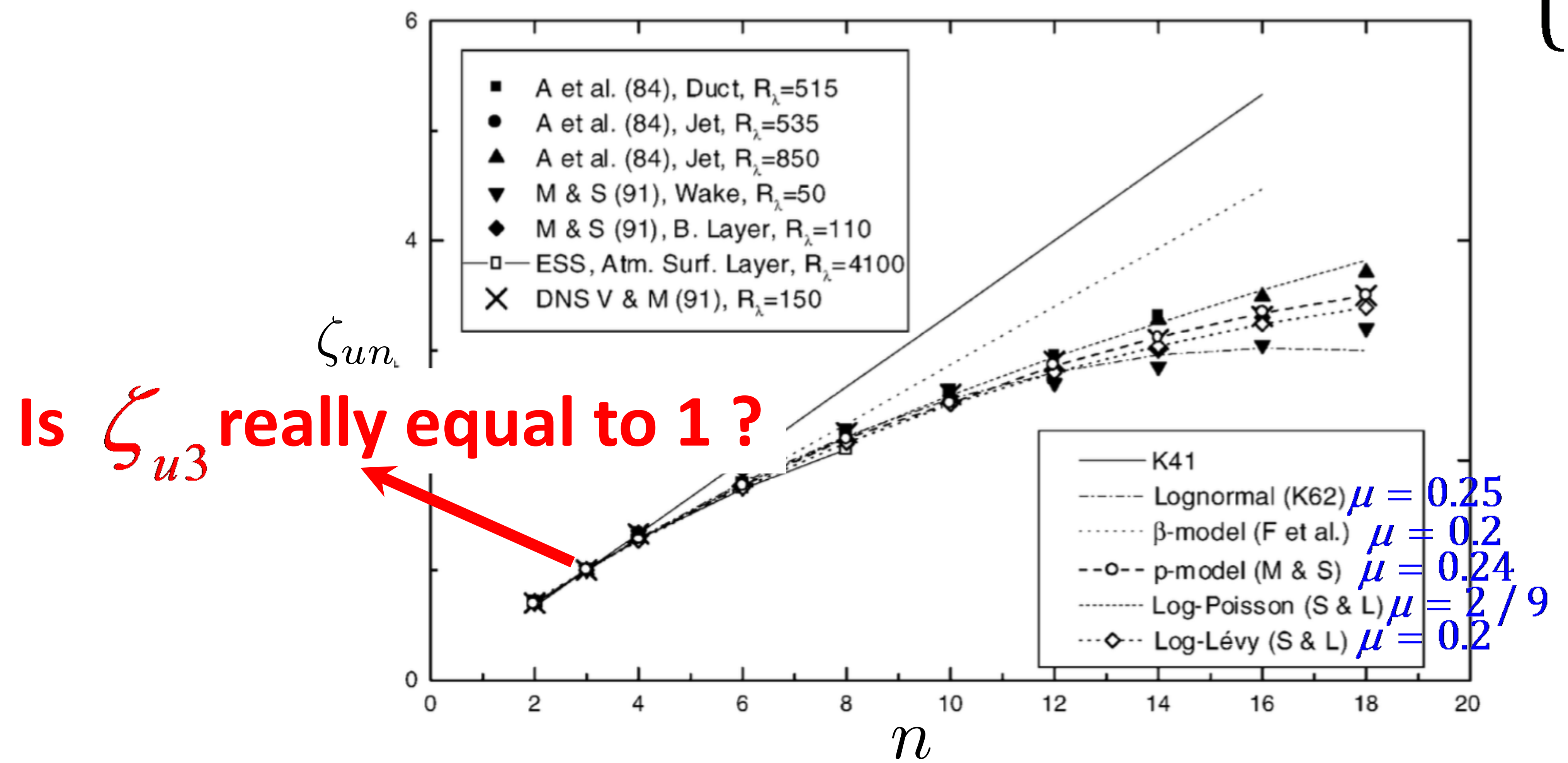
$$\Rightarrow \qquad -\frac{\overline{(\delta u)^3}}{u'^3} = \frac{4}{5} C_\epsilon \frac{r}{L}$$

with $\quad \overline{\epsilon} = C_\epsilon u'^3 / L$

i.e. the “4/5” law survives if $\quad C_\epsilon = 1$!

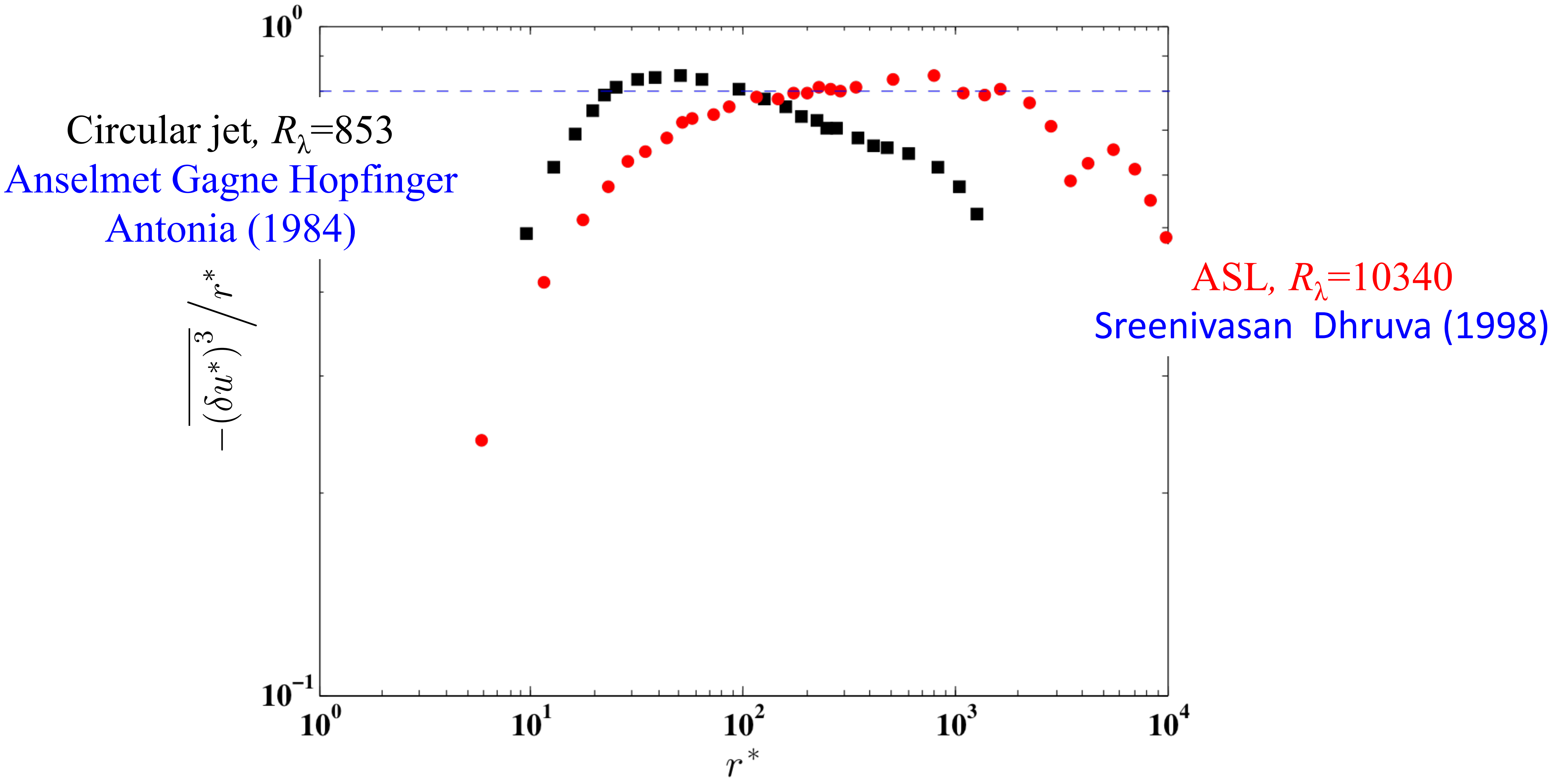
support for K62, **i.e. ‘anomalous’ scaling ?**

$$\overline{(\delta u)^n} \sim r^{\zeta_{un}}$$



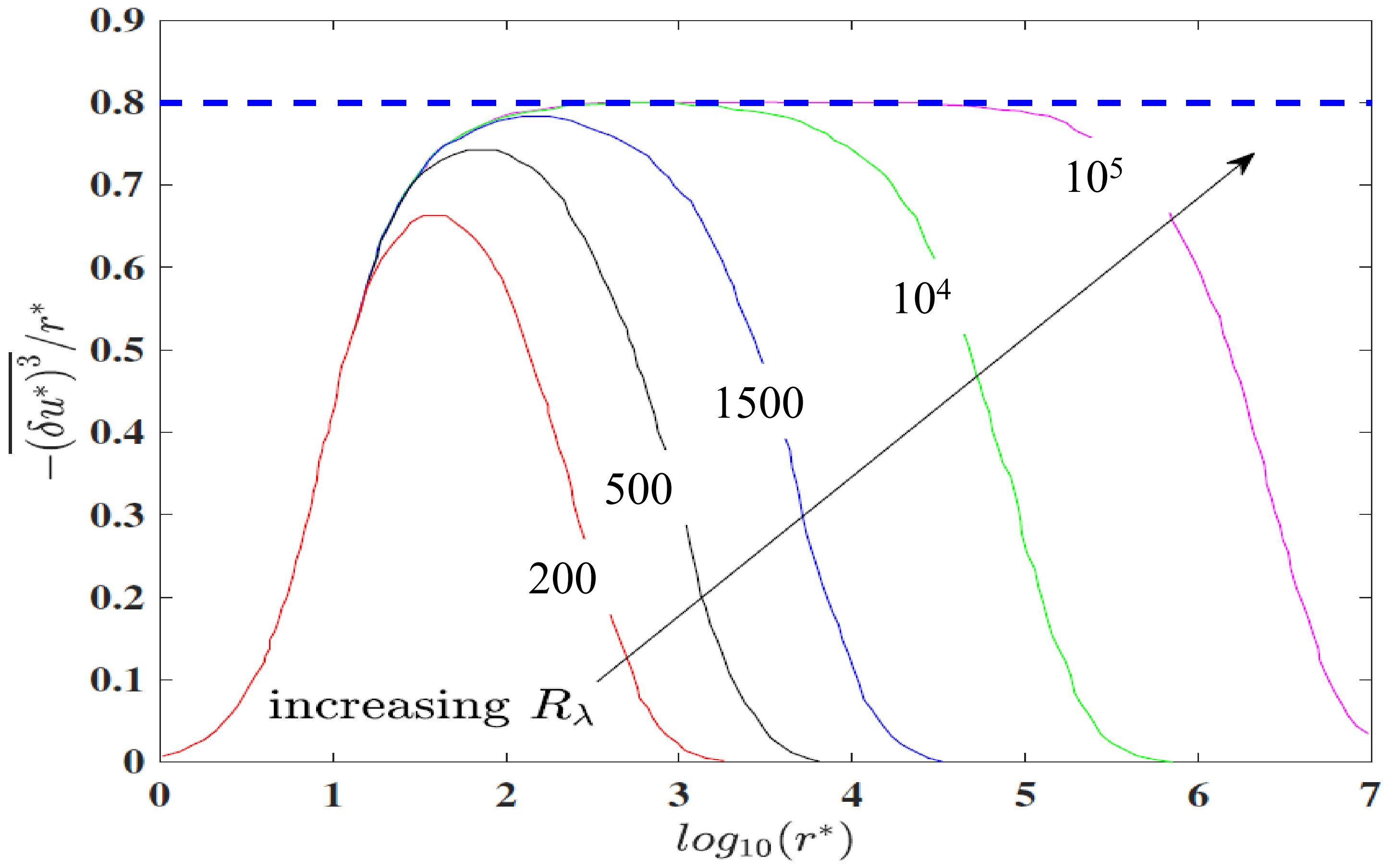
Anselmet Gagne Hopfinger Antonia 1984

Anselmet Antonia Danaila 2001



Circular jet, R_λ=853
Anselmet Gagne Hopfinger
Antonia (1984)
ASL, R_λ=10340
Sreenivasan Dhruva (1998)
$-\overline{(\delta u^*)^3}/r^*$
r^*
10^0
10^{-1}
10^0
10^1
10^2
10^3
10^4

QIAN JIAN 1997 1999



Estimation of non homogeneous LARGE SCALE TERM in various flows

Danaila Anselmet Zhou Antonia 1999 Danaila Anselmet Antonia 2002 Danaila Antonia Burattini 2004

e.g. s-b-s energy budget equation along the axis in the far field of an axisymmetric jet is

Tang Antonia Djenidi Danaila Zhou 2017

$$-\overline{(\delta u)^3} = \frac{4}{5}\bar{\varepsilon} r - 6\upsilon \frac{d}{dr}\overline{(\delta u)^2} + I_u$$

$$I_u = -\frac{3}{r^4}\int_0^r s^4 \left[U \frac{\partial \overline{(\delta u)^2}}{\partial x} + 2\left[\overline{(\delta u)^2} - \overline{(\delta v)^2}\right]\frac{\partial U}{\partial x}\right] ds$$

After dividing by u_K^3 and taking $r=\lambda$

$$-\left.\frac{\overline{(\delta u)^3}}{u_K^3}\right|_{r=\lambda} = T_\lambda$$

$$\frac{4}{5u_K^3}\bar{\varepsilon} r = \frac{4}{5}\lambda^*$$

$$\frac{6\nu}{u_K^3}\frac{\partial \overline{(\delta u)^2}}{\partial r} = 6\left.\frac{\partial \overline{(\delta u^*)^2}}{\partial r^*}\right|_{r=\lambda} = V_\lambda$$

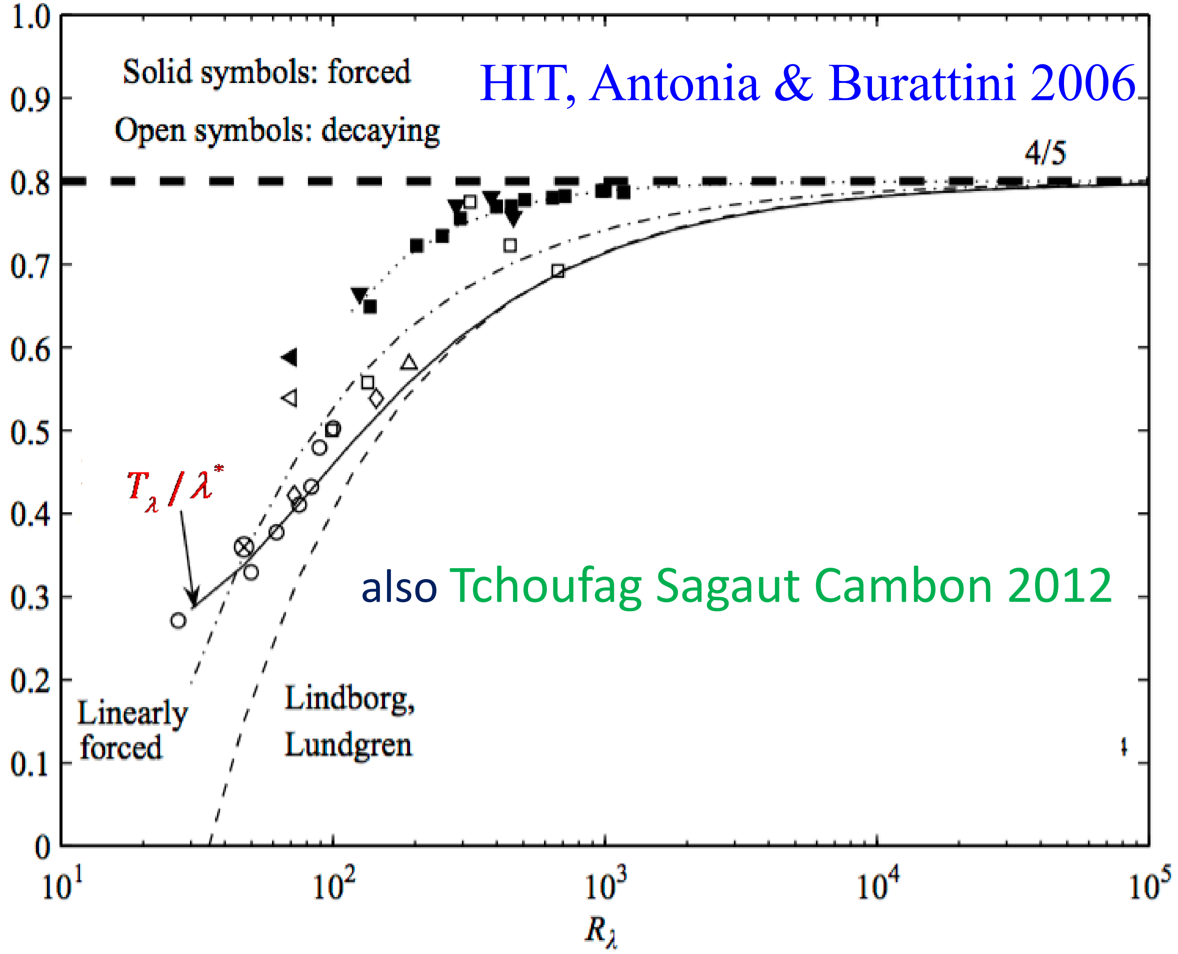
$$I_u(\lambda^*) = \left(\frac{3\sqrt{15}}{2+R}\right) R_\lambda^{-1}(\Gamma_1^* + 4\Gamma_2^* - 2\Gamma_3^*) r^{*-4}$$

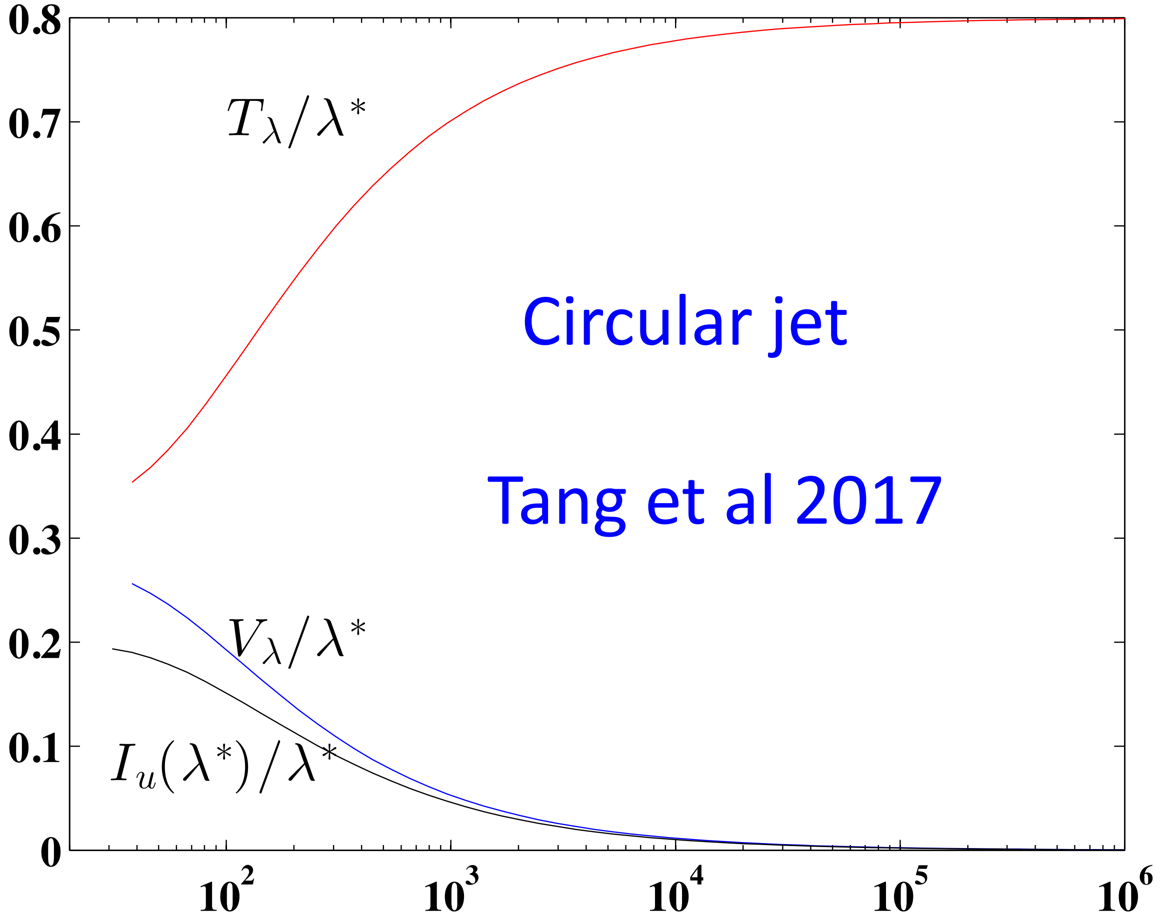
$$\Gamma_1^* = \int_0^{r^*} s^{*5} \frac{\partial \overline{(\delta u^*)^2}}{\partial r^*} ds^*, \Gamma_2^* = \int_0^{r^*} s^{*4} \overline{(\delta u^*)^2} ds^*,$$

$$\Gamma_3^* = \int_0^{r^*} s^{*4} \overline{(\delta v^*)^2} ds^*$$

$$I_u(\lambda^*) + T_\lambda + V_\lambda = \frac{4}{5}\lambda^*$$

A parameterized form of $\overline{(\delta u^*)^2}$ **Substitute into** $I_u(\lambda^*) + T_\lambda + V_\lambda = \frac{4}{5}\lambda^*$





$$\overline{(\delta u^*)^2} = \frac{r^{*2}(1 + r^*/L^*)^{-2/3}}{15\left(1 + (r^*/r_c^*)^2\right)^{2/3}}$$

$$r_c^* = (15C_{u2})^{3/4} \quad (C_{u2} = 2.0)$$

$$L^* = C_\varepsilon 15^{-3/4} R_\lambda^{3/2}$$

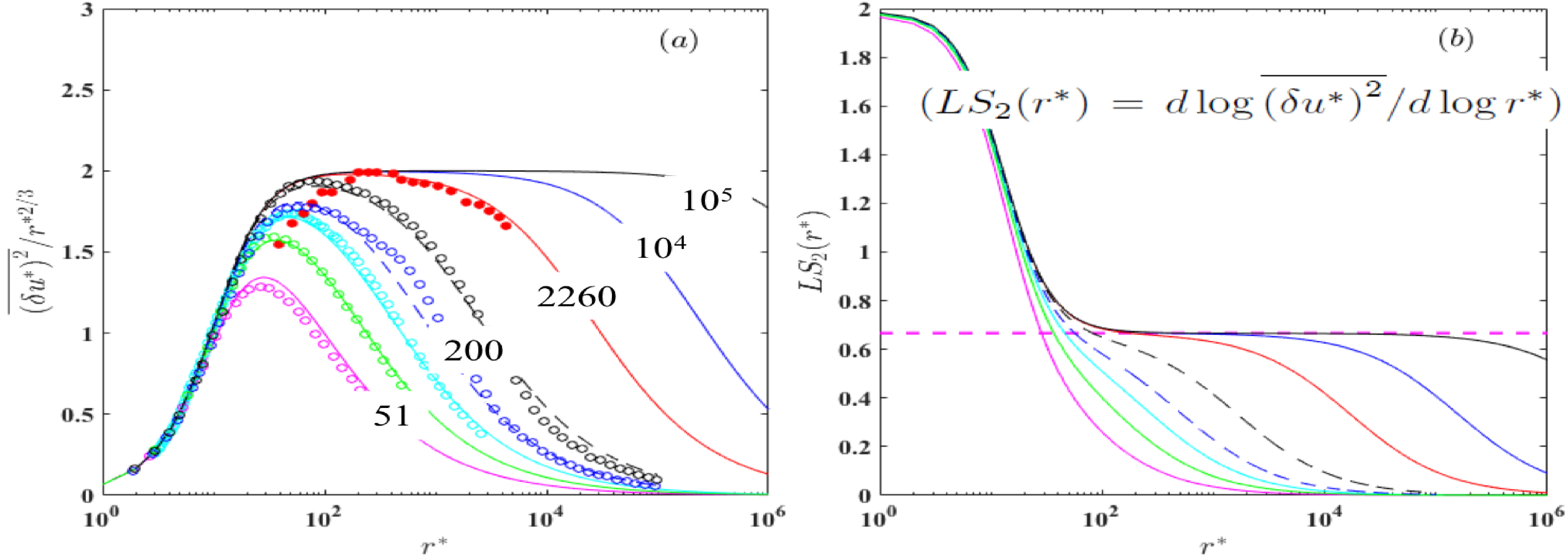
where $C_\epsilon = \bar{\varepsilon} L / u'^3 \approx 1.2$ for grid turbulence;

$C_\epsilon \approx 1.4$ for circular jet.

○, ○: circular jet

Other symbols: grid turbulence

Curves: model

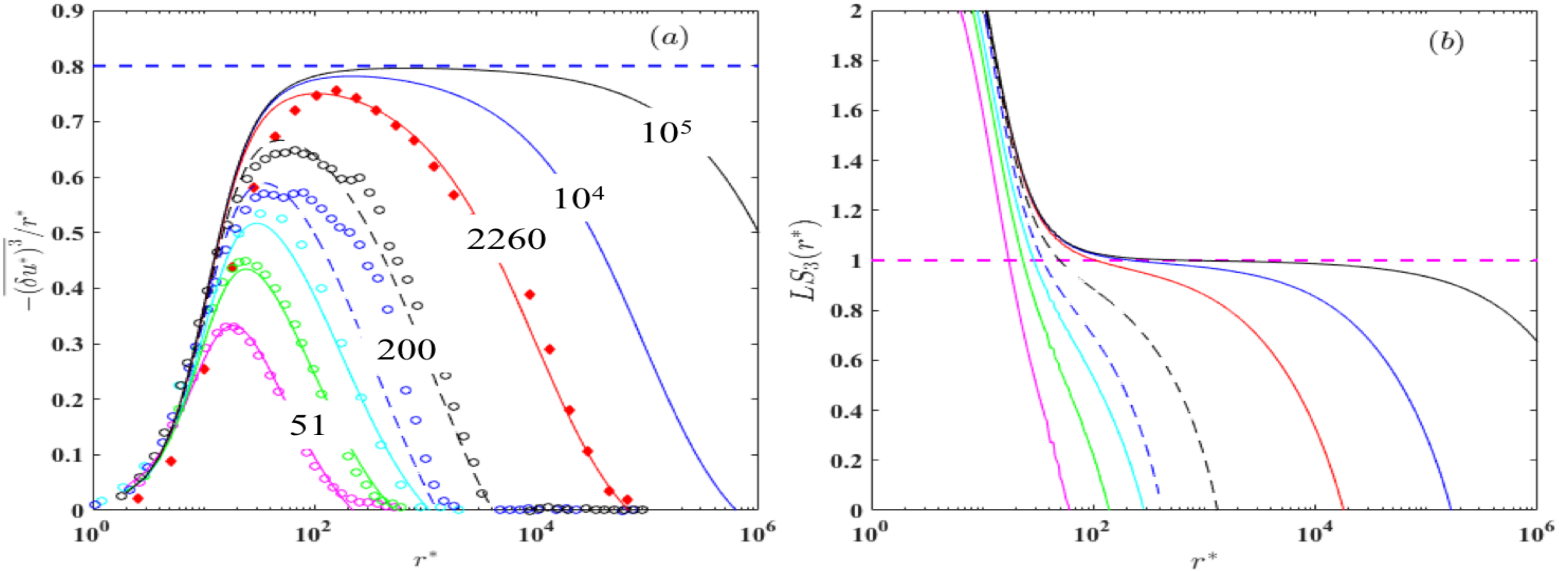


$$-\overline{(\delta u)^3} = \frac{4}{5}\bar{\epsilon} r - 6\nu \frac{\partial \overline{(\delta u)^2}}{\partial r} + I_u \quad \text{where} \quad I_u = \frac{3}{r^4}\int_0^r s^4 \frac{\partial \overline{(\delta u)^2}}{\partial t} ds \quad \text{for grid turbulence}$$

$$I_u = \frac{3}{r^4}\int_0^r s^4 \left[U \frac{\partial \overline{(\delta u)^2}}{\partial x} + 2\left[\overline{(\delta u)^2} - \overline{(\delta v)^2}\right]\frac{\partial U}{\partial x}\right] ds \quad \text{for circular jet}$$

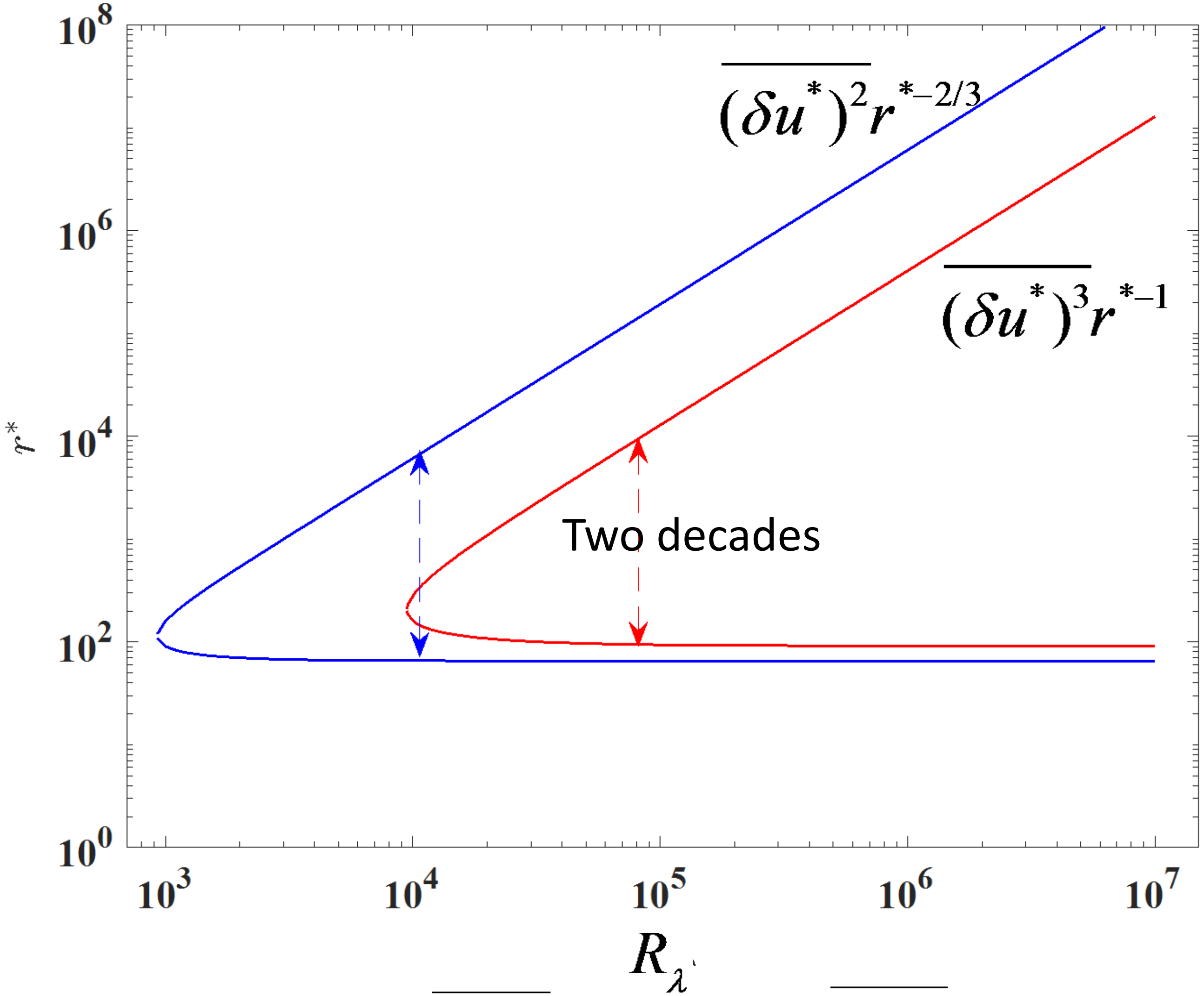
○, ○: circular jet
Other symbols: grid turbulence
Curves: SBS

$$(LS_3(r^*) = d \log \overline{(\delta u^*)^3} / d \log r^*)$$

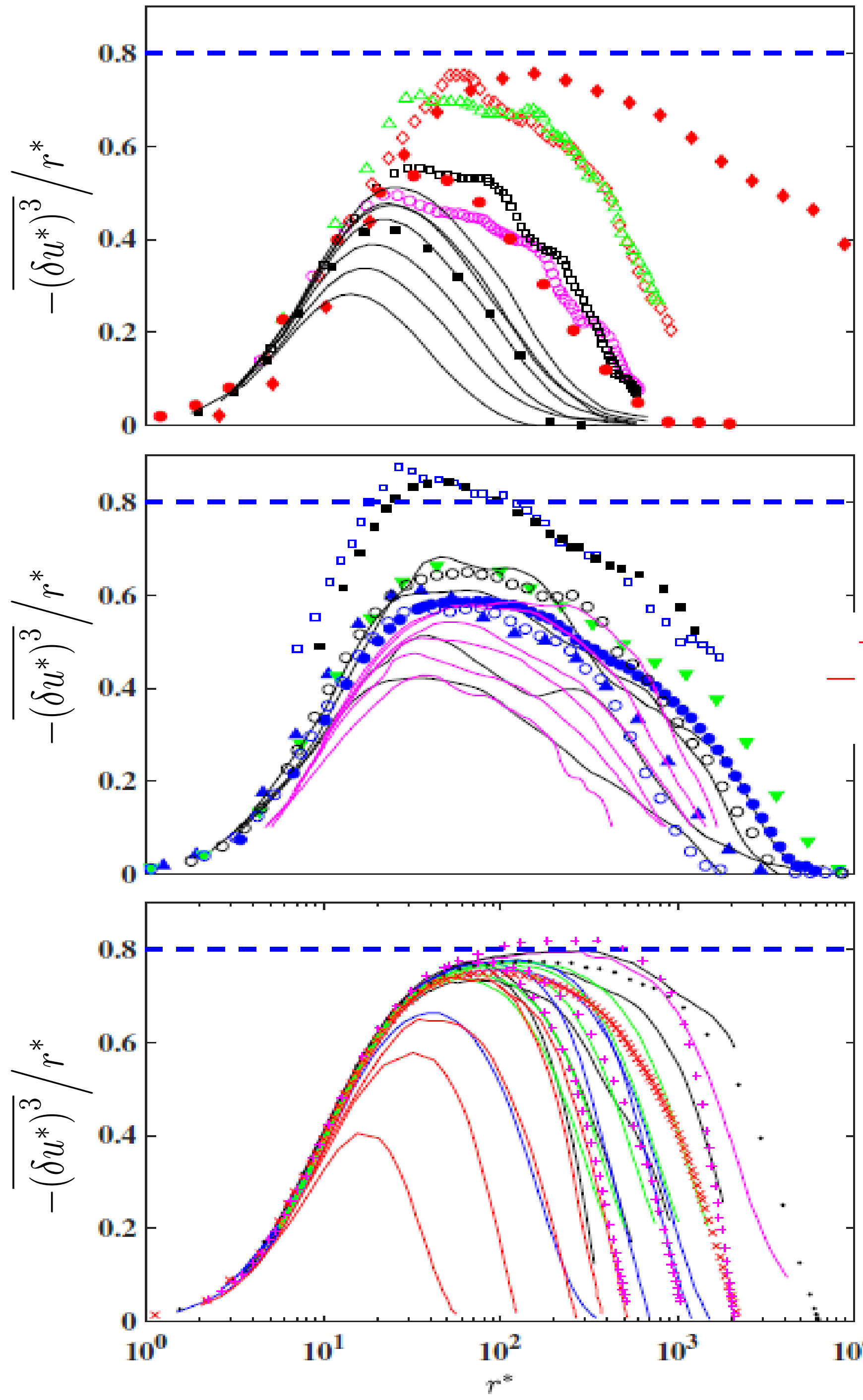


$$\overline{(\delta u^*)^2} = \frac{r^{*2}(1 + r^*/L^*)^{-2/3}}{15\left(1 + (r^*/r_c^*)^2\right)^{2/3}}$$

$$-\overline{(\delta u)^3} = \frac{4}{5}\bar{\epsilon} r - 6\nu \frac{\partial \overline{(\delta u)^2}}{\partial r} + \frac{3}{r^4}\int_0^r s^4 \frac{\partial \overline{(\delta u)^2}}{\partial t} ds$$



The range of $r*$ over which $\overline{(\delta u^*)^2} r^{*-2/3}$ and $\overline{(\delta u^*)^3} r^{*-1}$ depart from 2 and 4/5 respectively by no more than 2.5%



“grid” turbulence

$$-\overline{(\delta u)^3} + 6\nu \frac{\partial \overline{(\delta u)^2}}{\partial r} - \frac{3}{r^4}\int_0^r s^4 \frac{\partial \overline{(\delta u)^2}}{\partial t} ds = \frac{4}{5}\overline{\varepsilon} r$$

axis of circular jet (far field)

$$-\overline{(\delta u)^3} + 6\nu \frac{\partial}{\partial r}\overline{(\delta u)^2} - \frac{3}{r^4}\int_0^r s^4 \left[U \frac{\partial \overline{(\delta u)^2}}{\partial x} + 2\left[\overline{(\delta u)^2} - \overline{(\delta v)^2}\right] \frac{\partial U}{\partial x} \right] ds = \frac{4}{5}\overline{\varepsilon} r$$

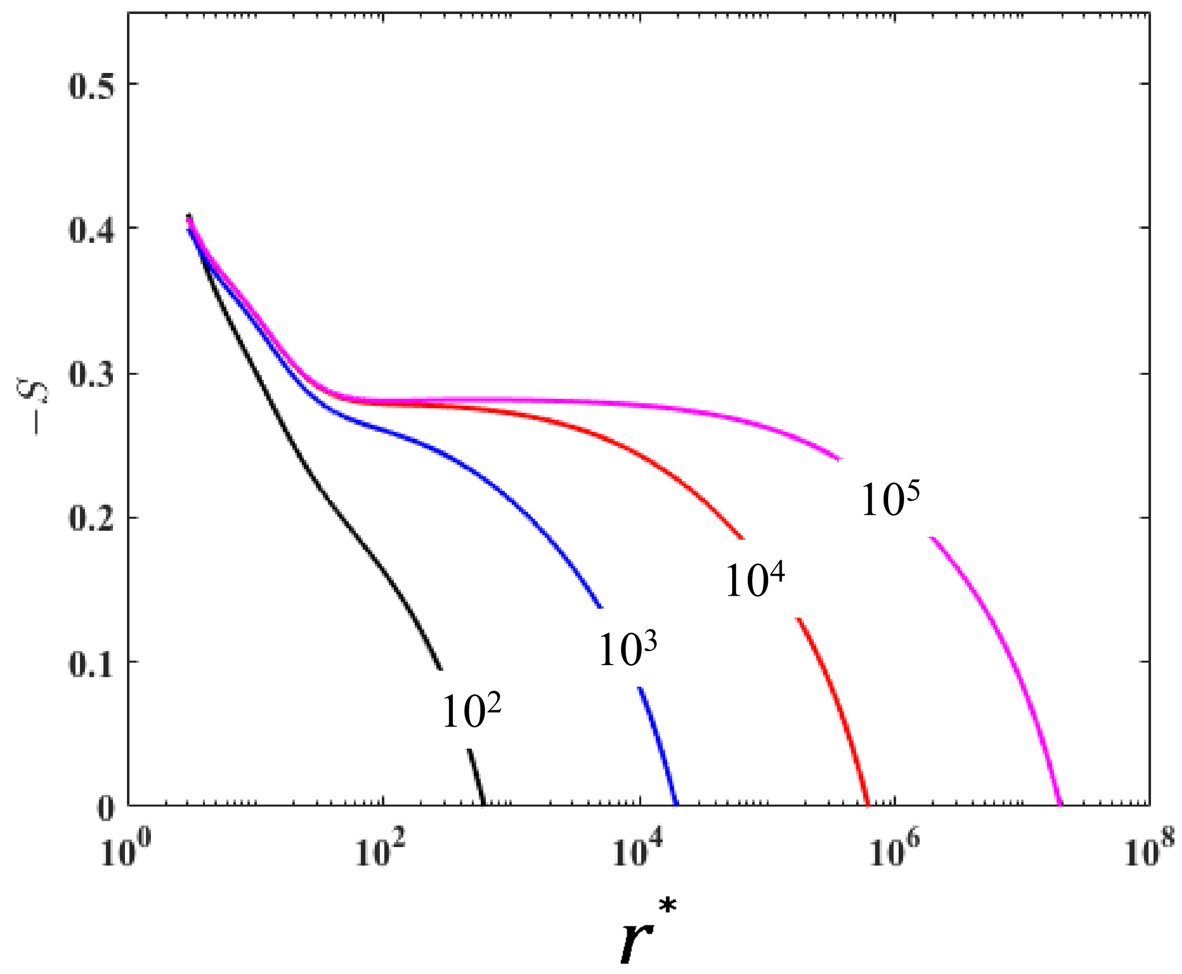
stationary forced periodic box turbulence DNS

$$-\overline{(\delta u)^3} + 6\nu \frac{\partial}{\partial r}\overline{(\delta u)^2} - \frac{2}{21}\varepsilon_{in}(k_e r)^2 r = \frac{4}{5}\overline{\varepsilon} r$$

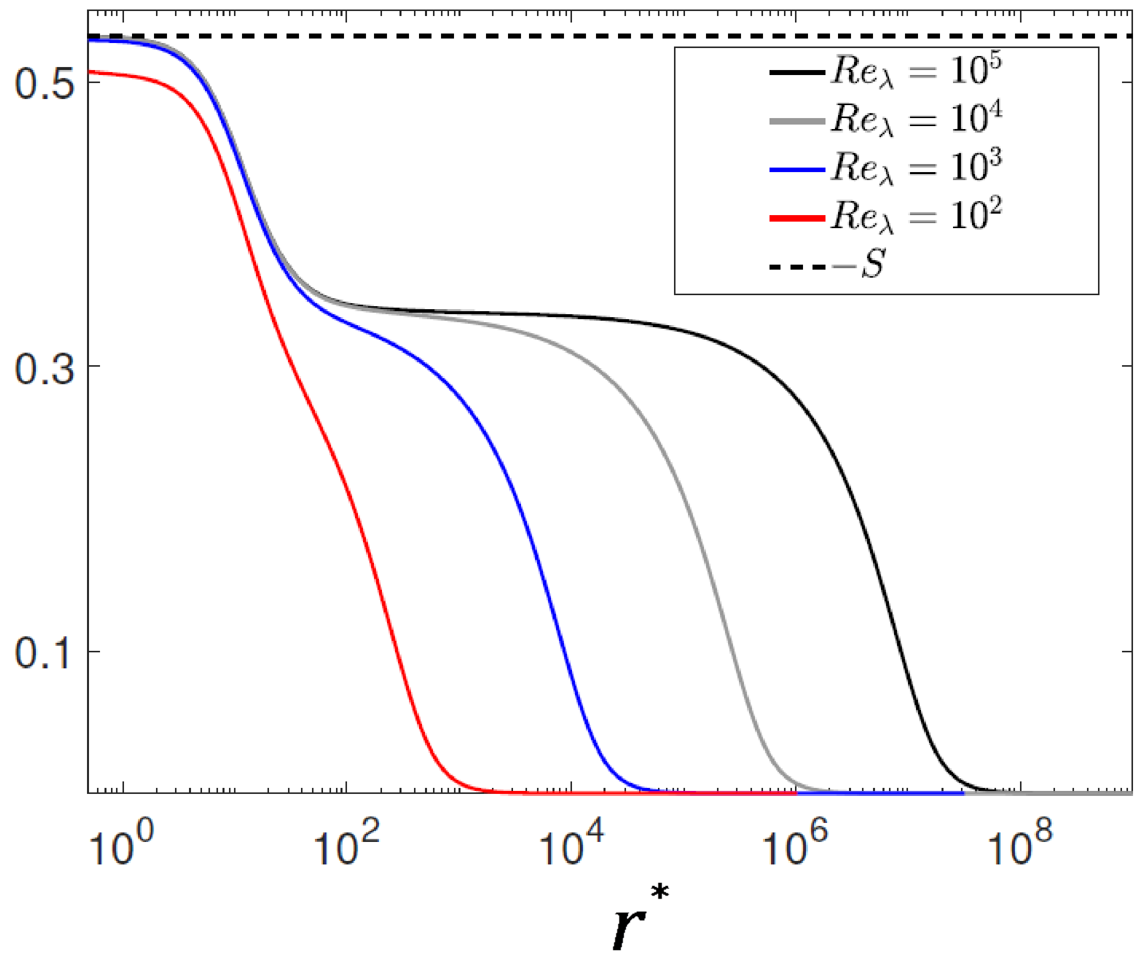
The forcing term is that used by Fukayama et al (2000) and may differ from DNS to DNS.

$$-S = \overline{(\delta u)^3} / \overline{(\delta u)^2}^{3/2}$$

(Decaying HIT)



Using s-b-s energy budget equation

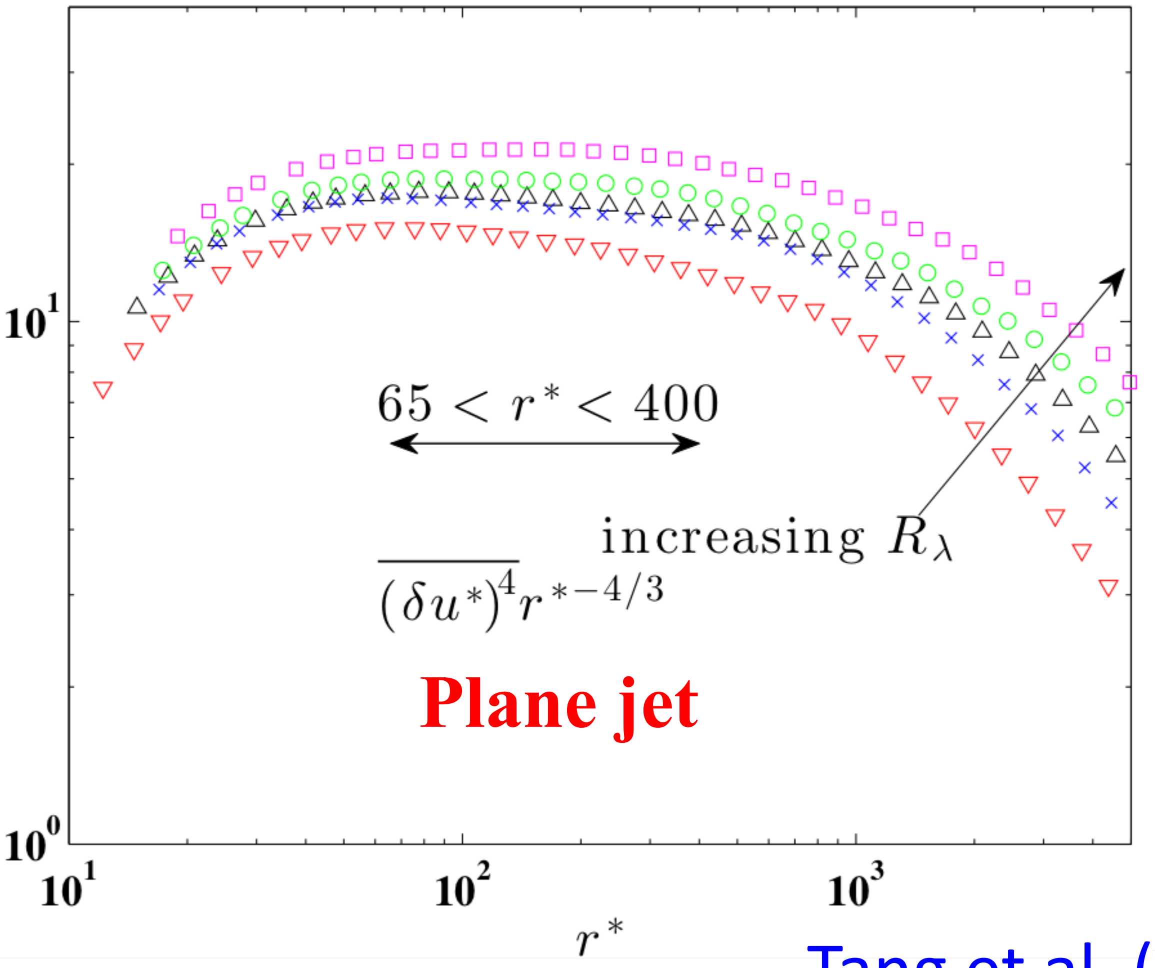


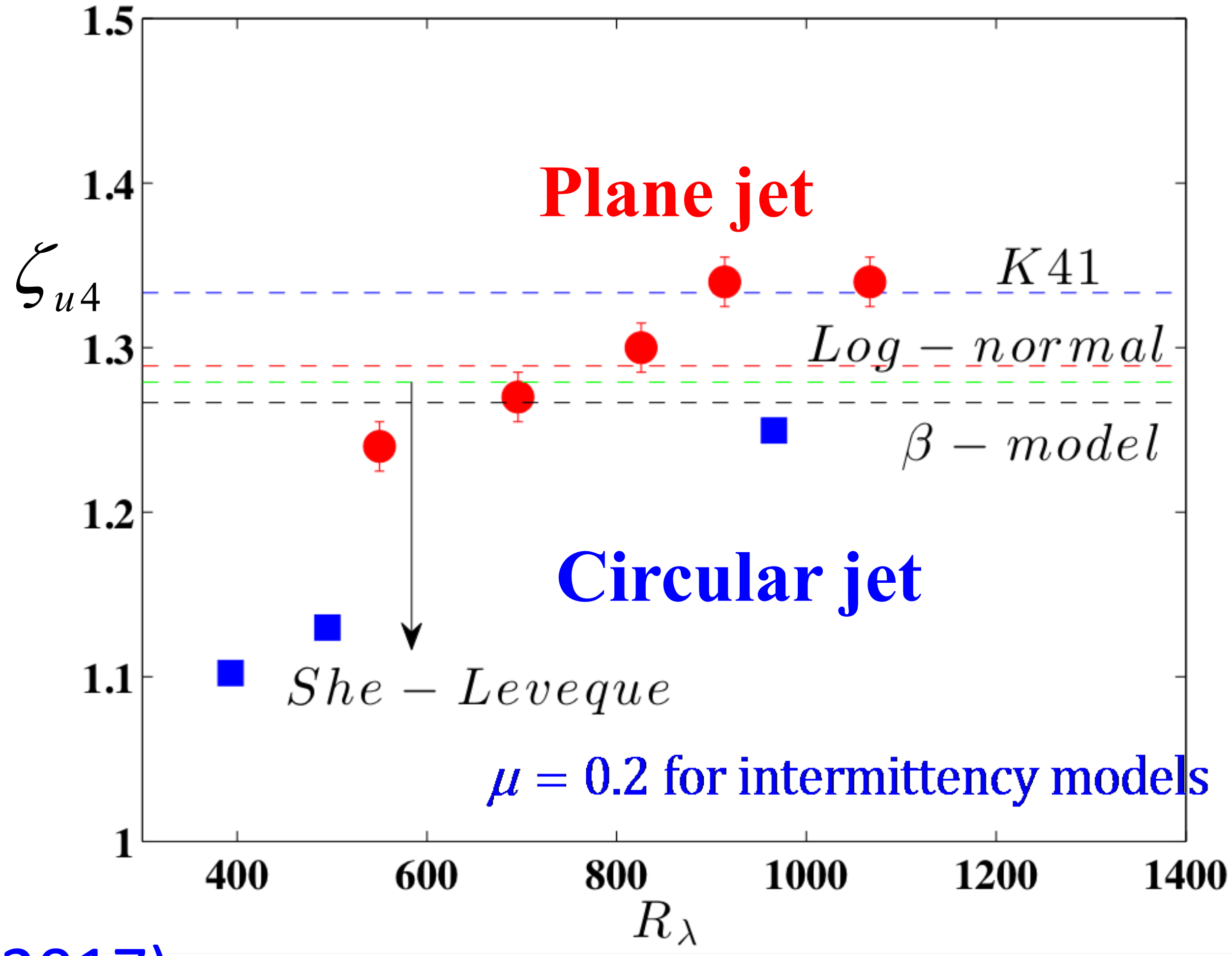
EDQNM (Meldi 2019, priv com)

The approach towards a plateau is consistent with scale-invariance at large Re (Djenidi Antonia Tang 2019)

Disappearance of the "anomaly"?

FRN effect on $\overline{(\delta u^*)^4}$

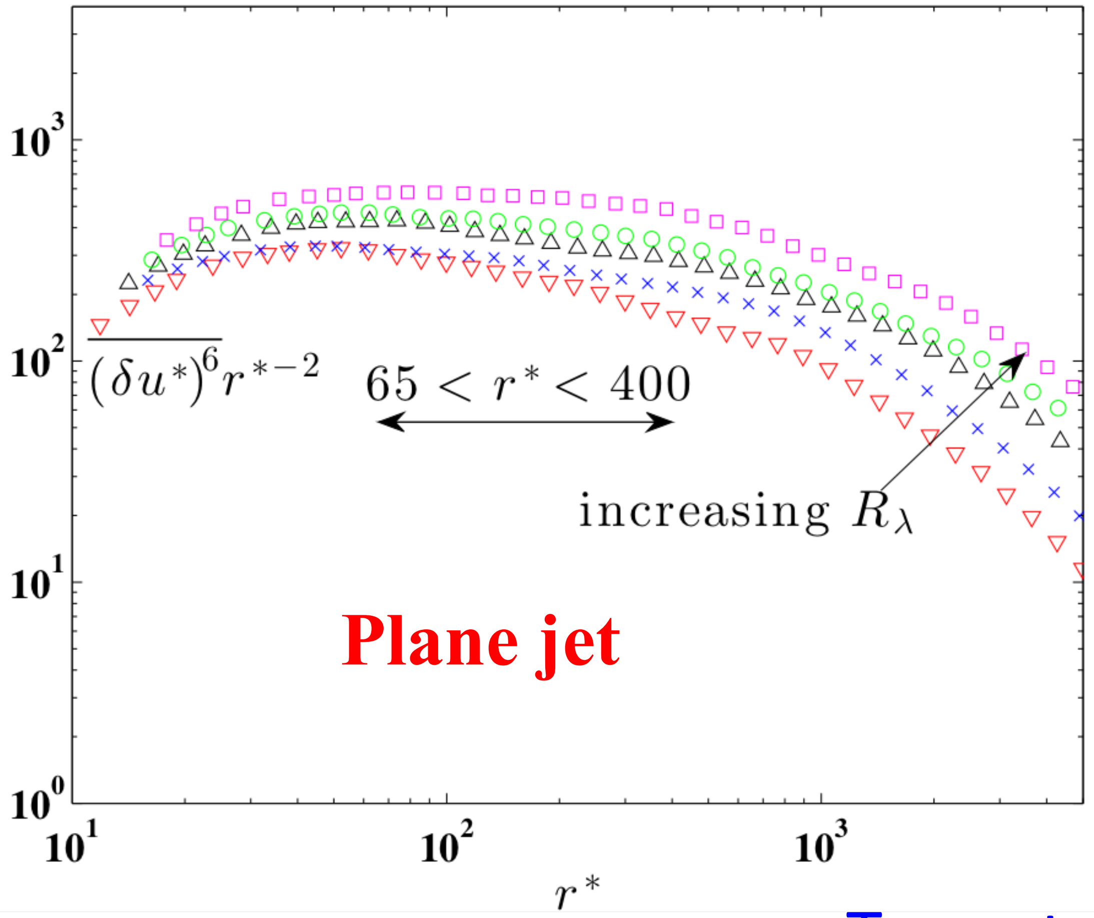


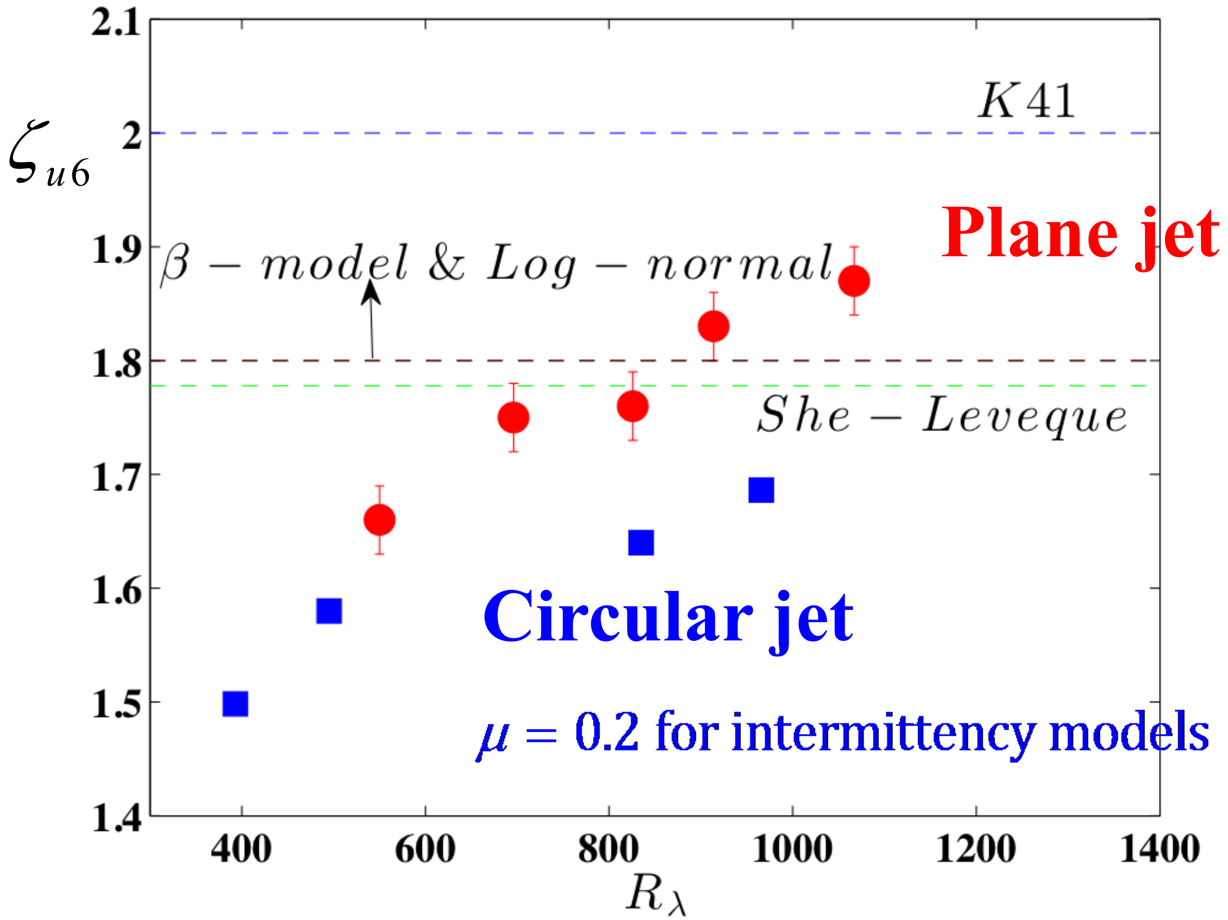


Tang et al. (2017)

Disappearance of the "anomaly"?

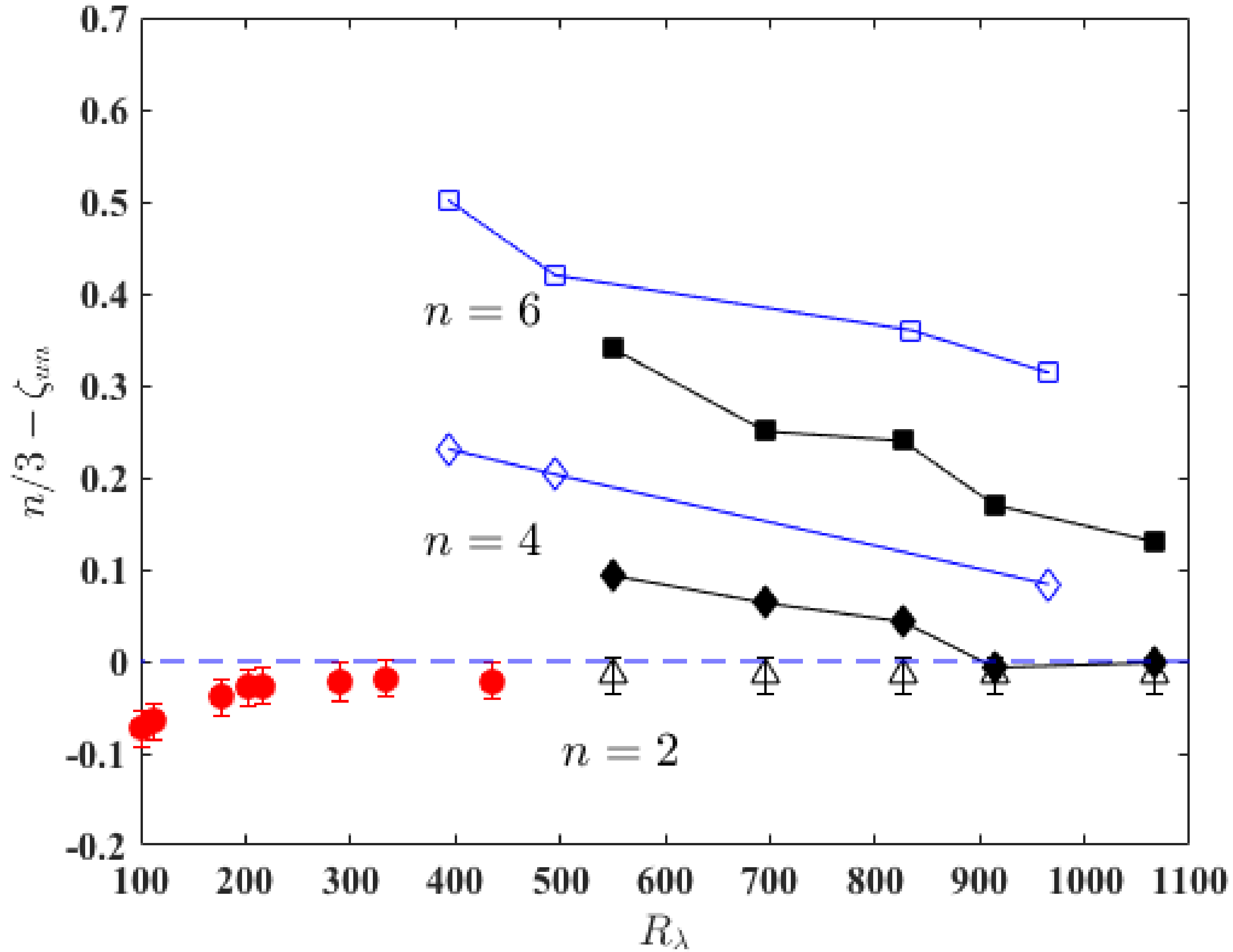
FRN effect on $\overline{(\delta u^*)^6}$





Tang et al. (2017)

Disappearance of the "anomaly"?



Antonia et al. (2017)
Tang et al. (2017)
McComb et al (2014)

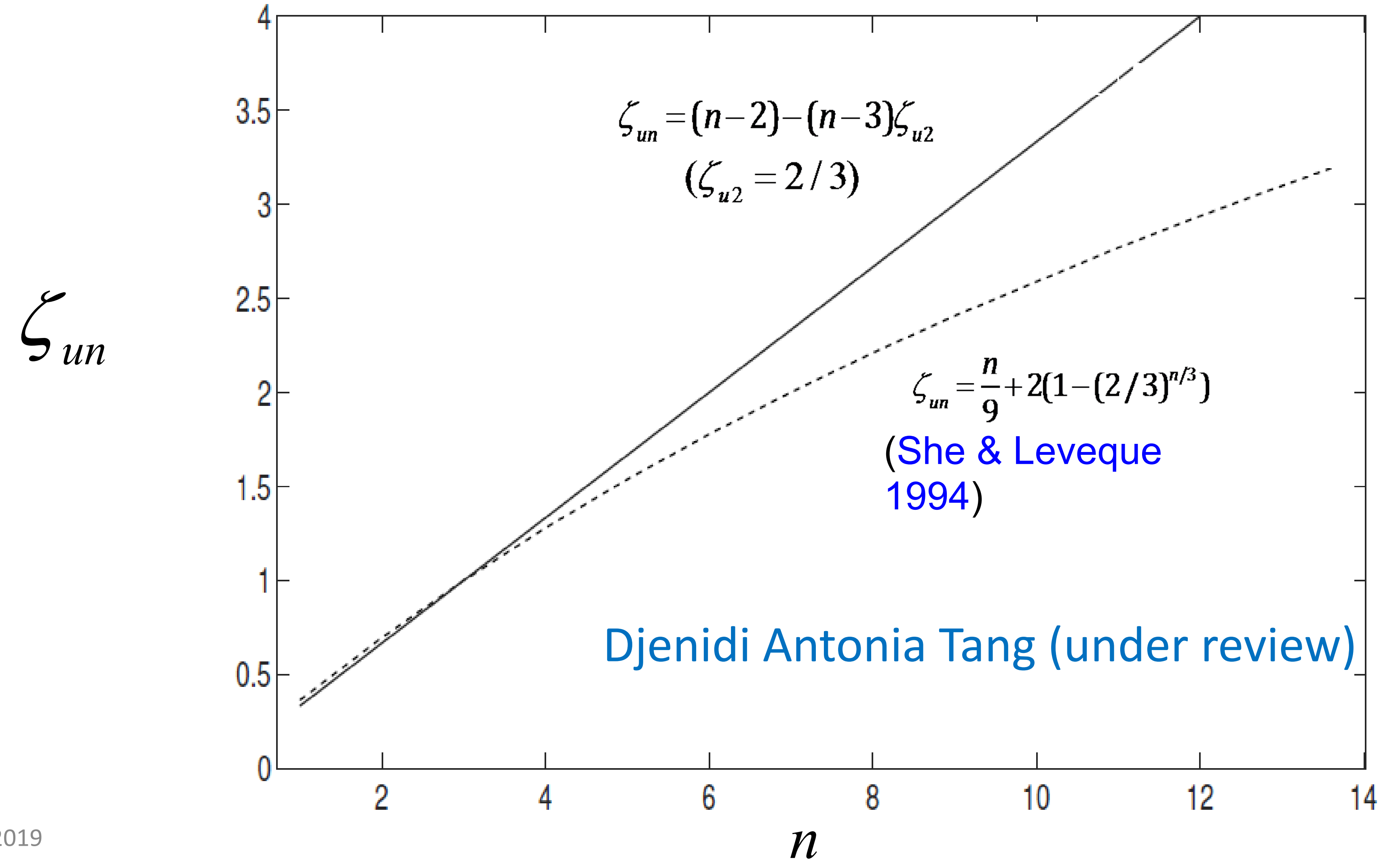
Blue symbols: circular jet
Black symbols: plane jet
Red symbols: box turbulence

Larger values of R_λ are needed

Assume: $\overline{(\delta u)^n} = A_n (\frac{r}{L})^{\zeta_{un}}$ and apply Cauchy-Schwarz inequality: $|\overline{(\delta u)^3}| = \frac{4}{5}\bar{\epsilon} r = |\overline{(\delta u)^2 (\delta u)}| \leq \overline{(\delta u)^4}^{1/2}\, \overline{(\delta u)^2}^{1/2}$

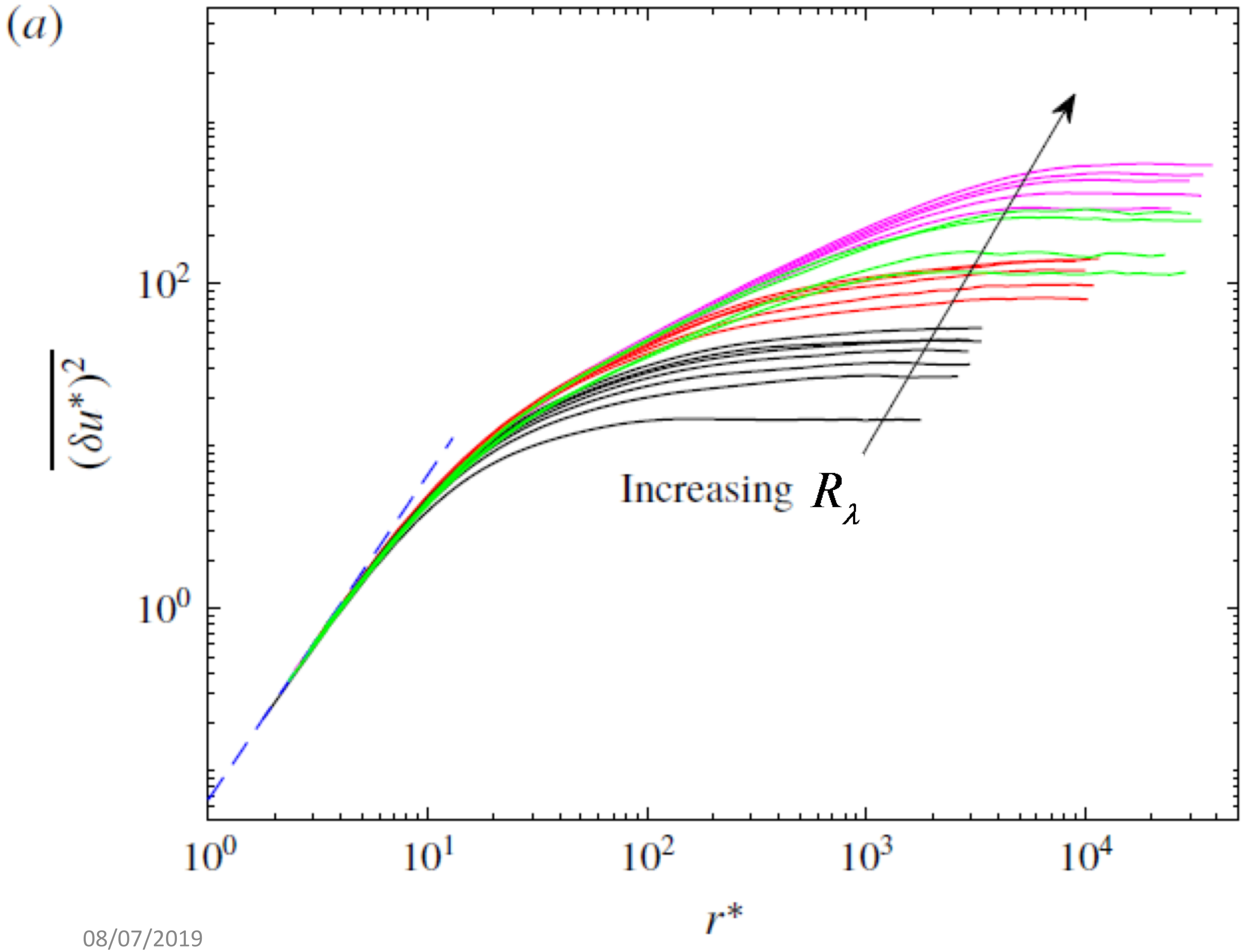
$\Rightarrow$ $\zeta_{u4} = 2 - \zeta_{u2}$ Also $A_2 A_4 \geq A_3^2$

In general: $\zeta_{un} = (n-2) - (n-3)\zeta_{u2}$



Kolmogorov scaling

Djenidi Antonia Tang 2019



Pink Plane Jet

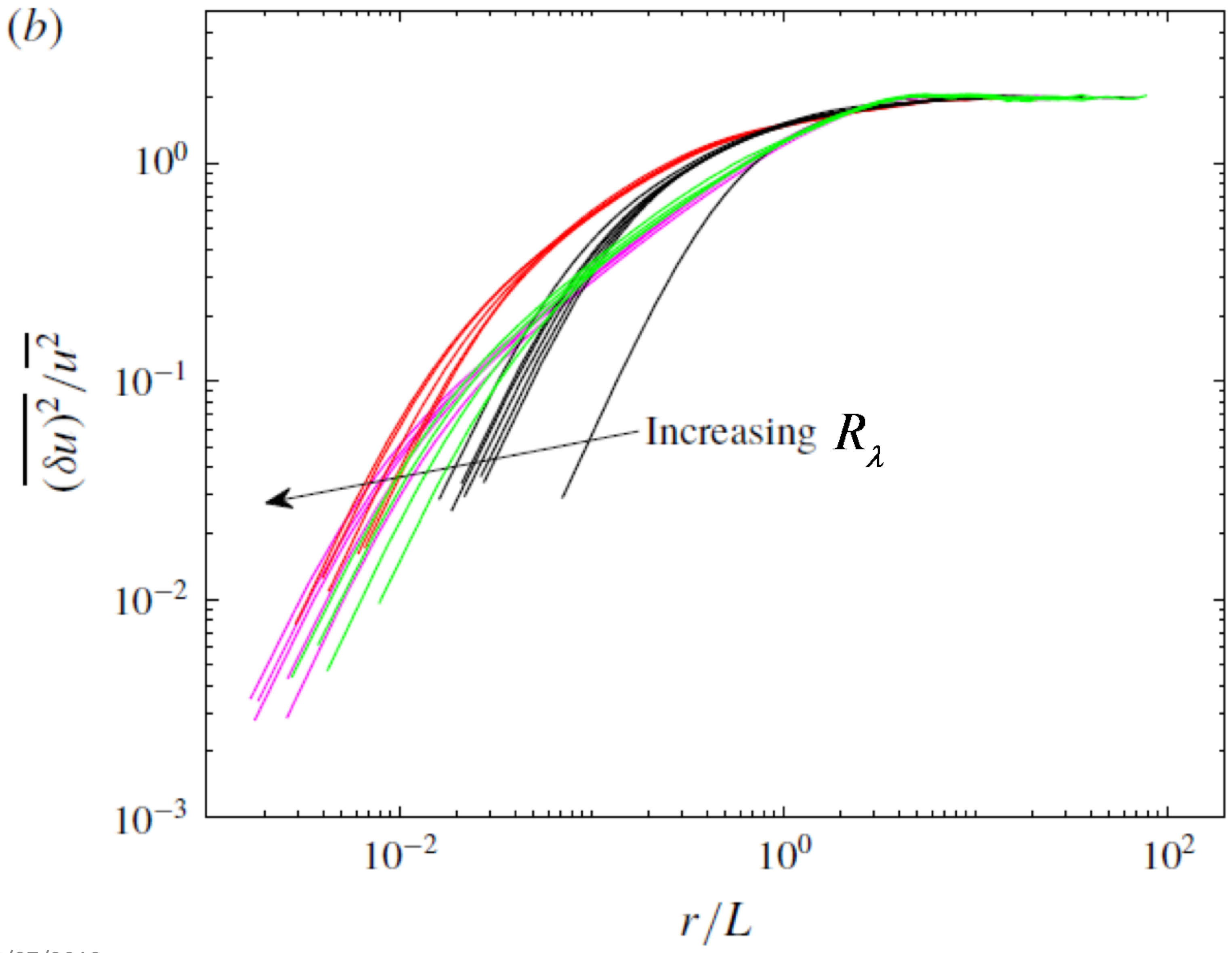
Green Circular Jet

Red Wake

Black Grid Turbulence

"External Scaling"

Djenidi Antonia Tang 2019



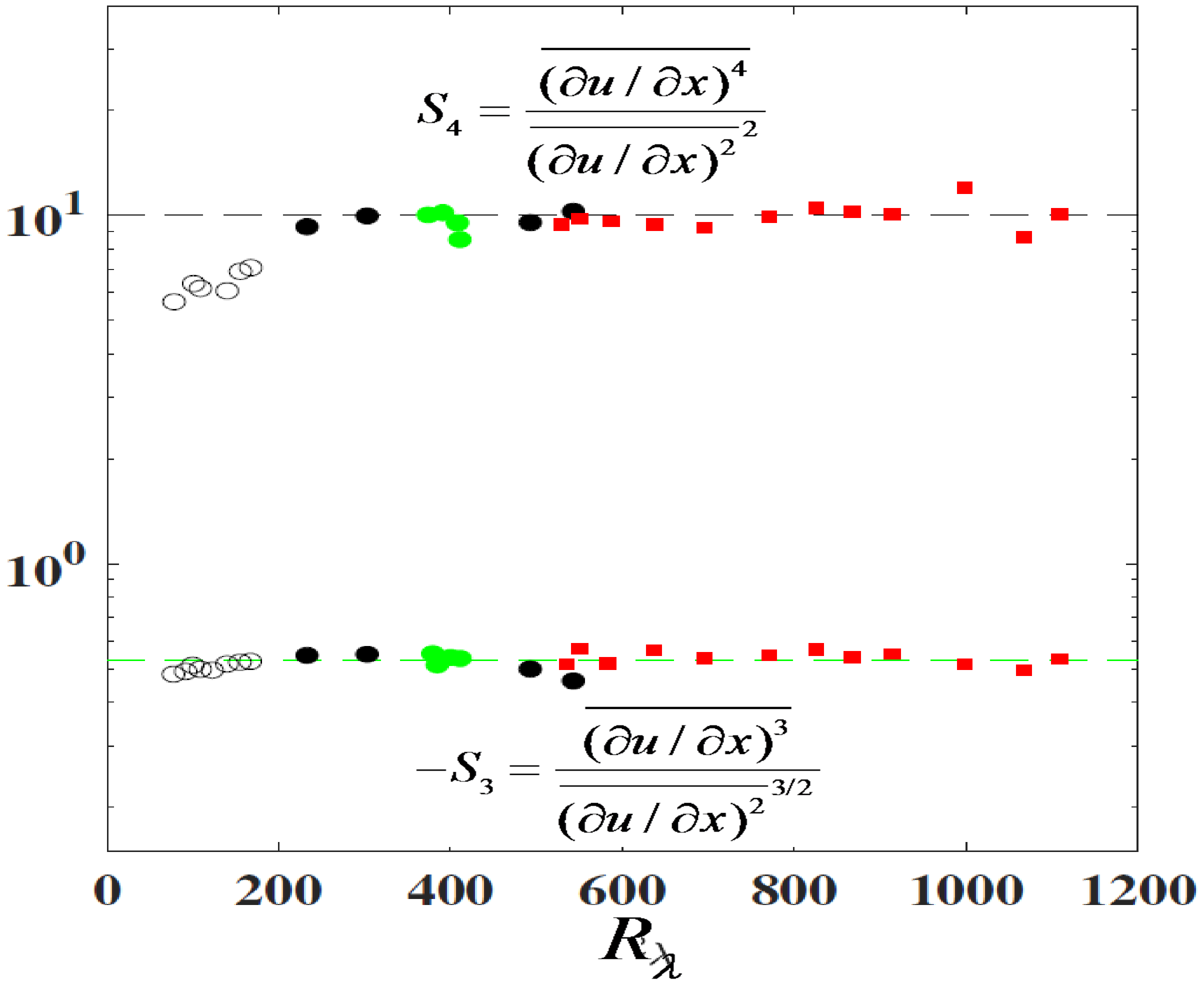
Pink Plane Jet

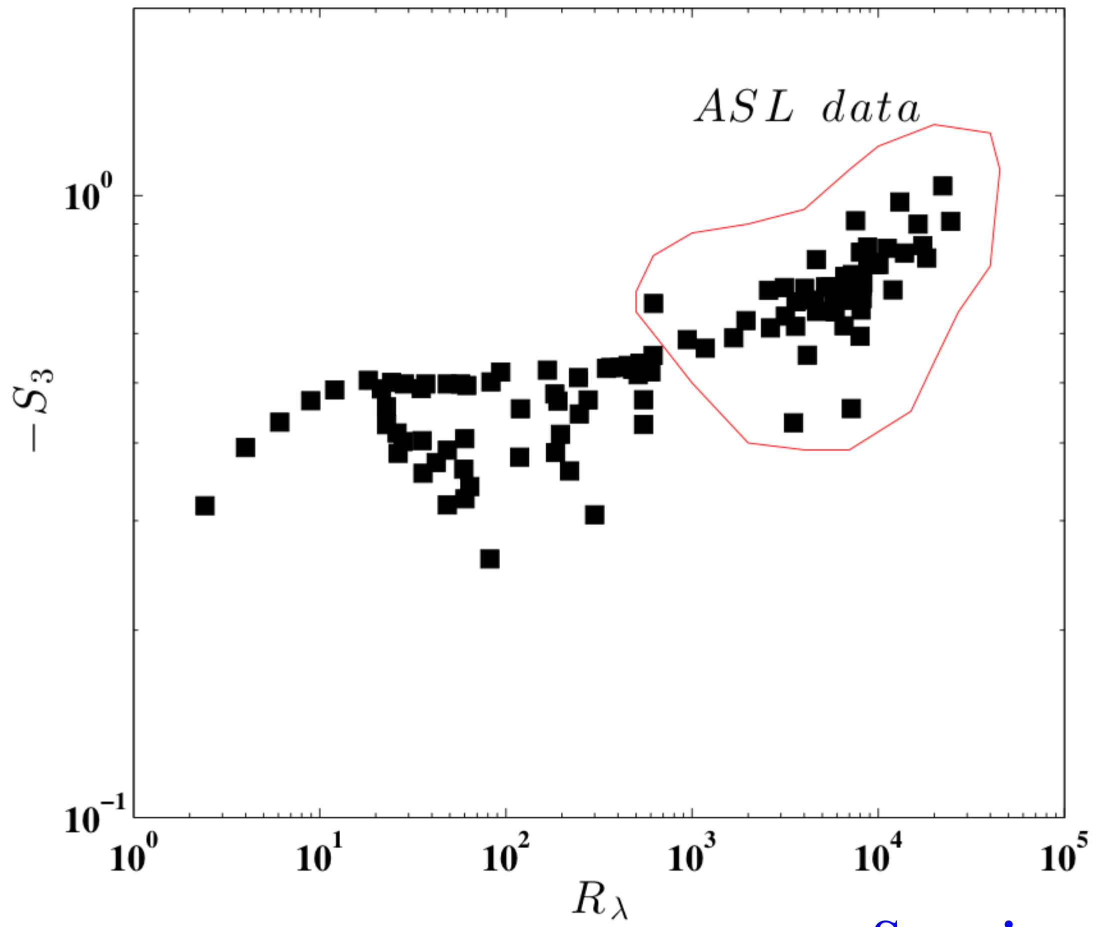
Green Circular Jet

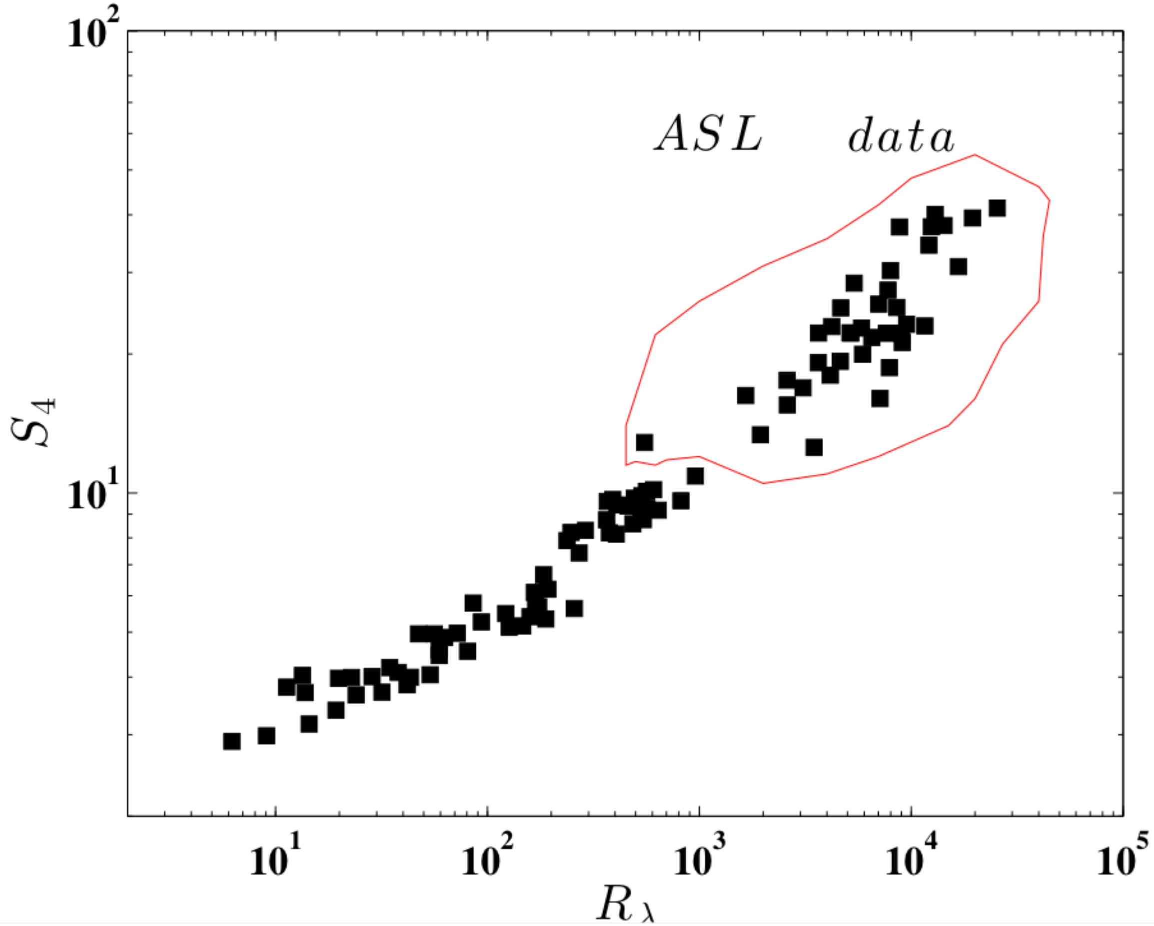
Red Wake

Black Grid Turbulence

Skewness (S_3) and Flatness (S_4) factor of $\partial u/\partial x$ in plane and circular jets

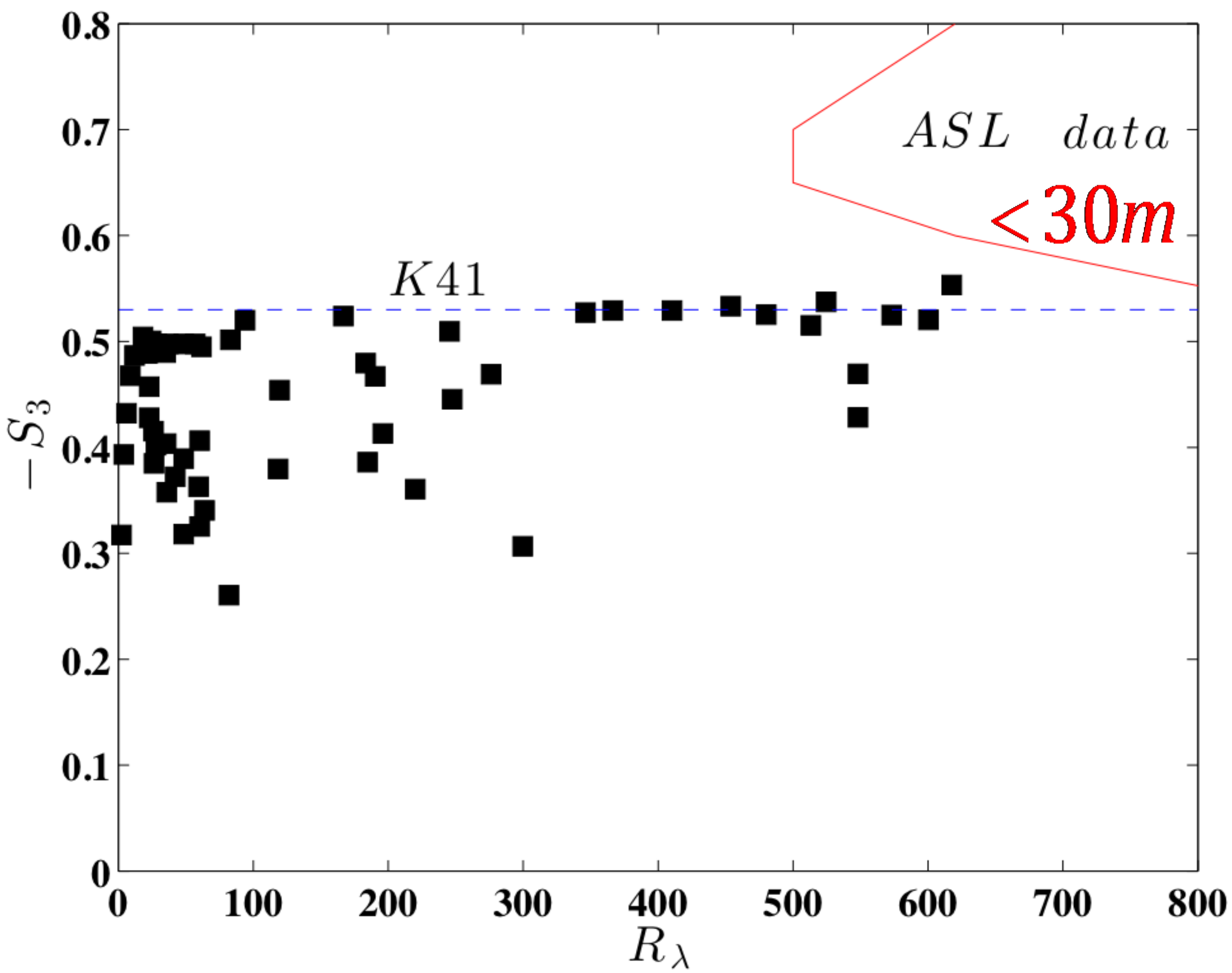


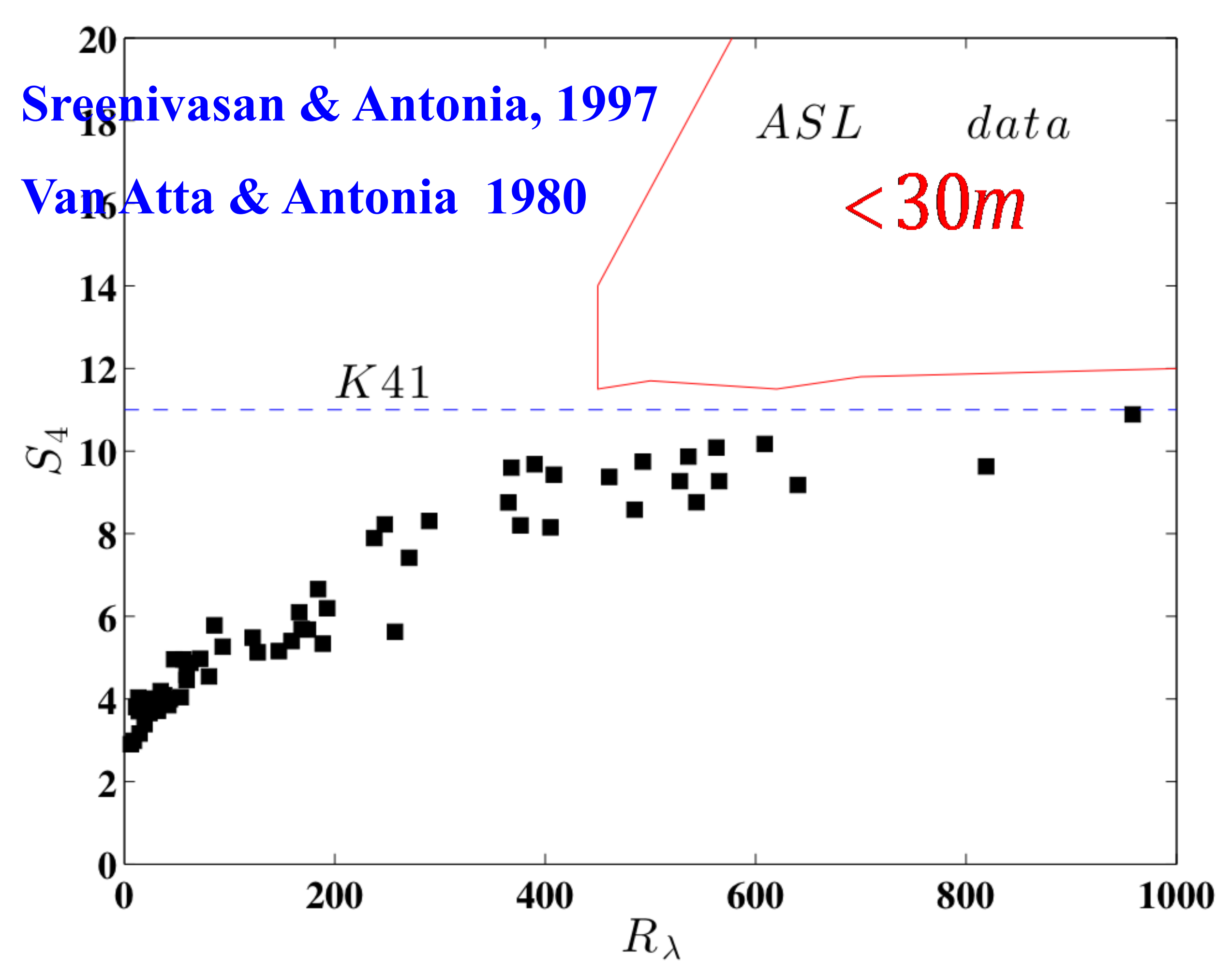




Sreenivasan & Antonia, 1997

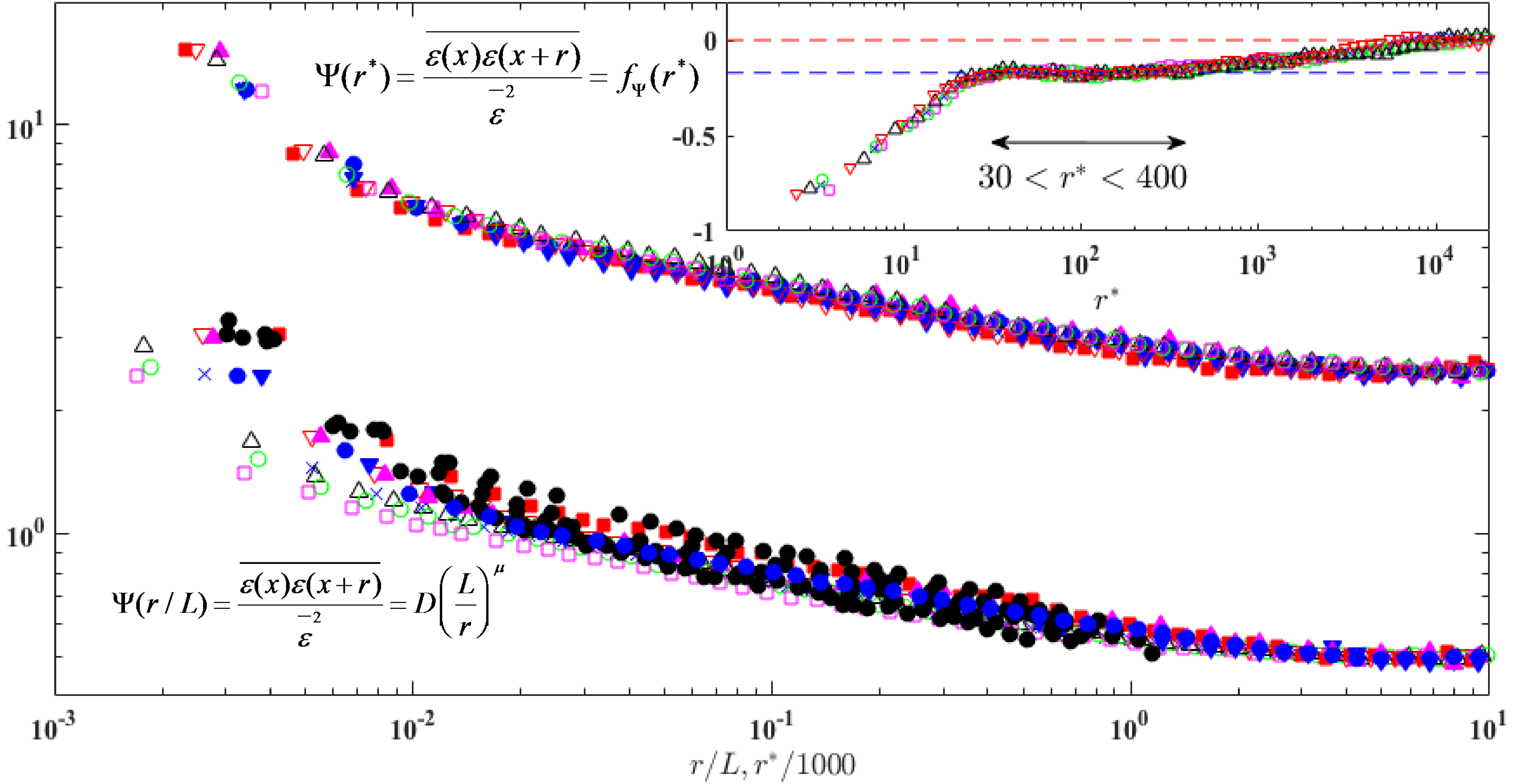
Van Atta & Antonia 1980





Lab data on linear axes

Tang Antonia Djenidi Zhou 2019



Conclusion 1

The KH equation has led to several major results:

- the “second hypothesis” (K41) has yet to be validated. Larger values of R_λ are required (the FRN effect becomes more important as n increases)
- The ‘anomalous’ scaling seems to be an FRN effect, in particular

$$\zeta_{u3} \neq 1$$

Conclusion 2

Guy Beart (circa 1976 ?)

Tourbillonnaire

Deux pas en avant (K41 ?)

Et quatre pas en arriere (K62 ?)

16 years ago in Newcastle

6 years ago in Rouen

March 2019 in Shenzhen


DROIT AU BUT
ALLEZ l'OM !
DROIT AU BUT